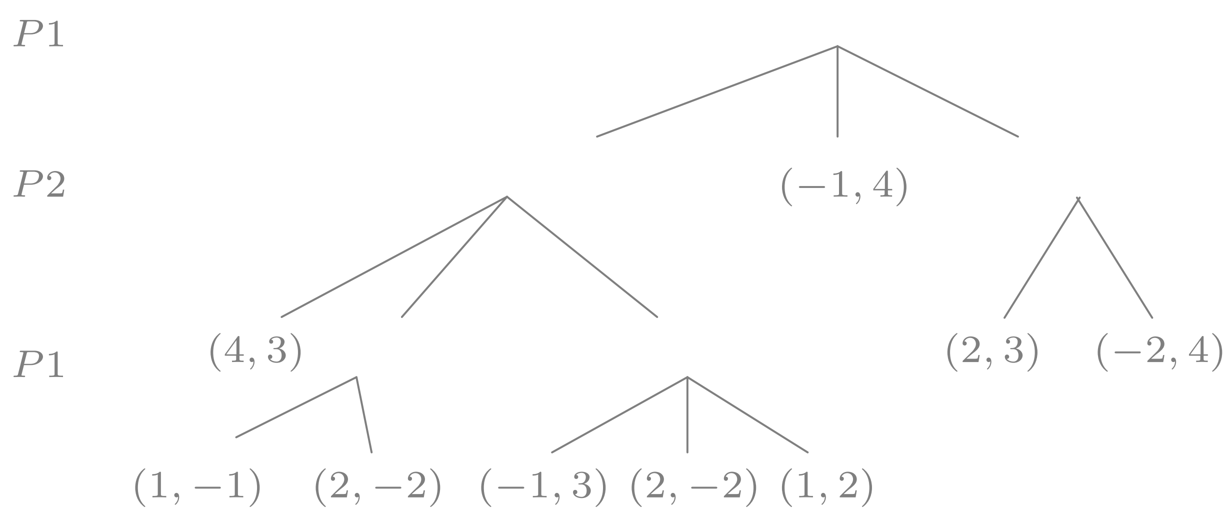
Minimax algorithm

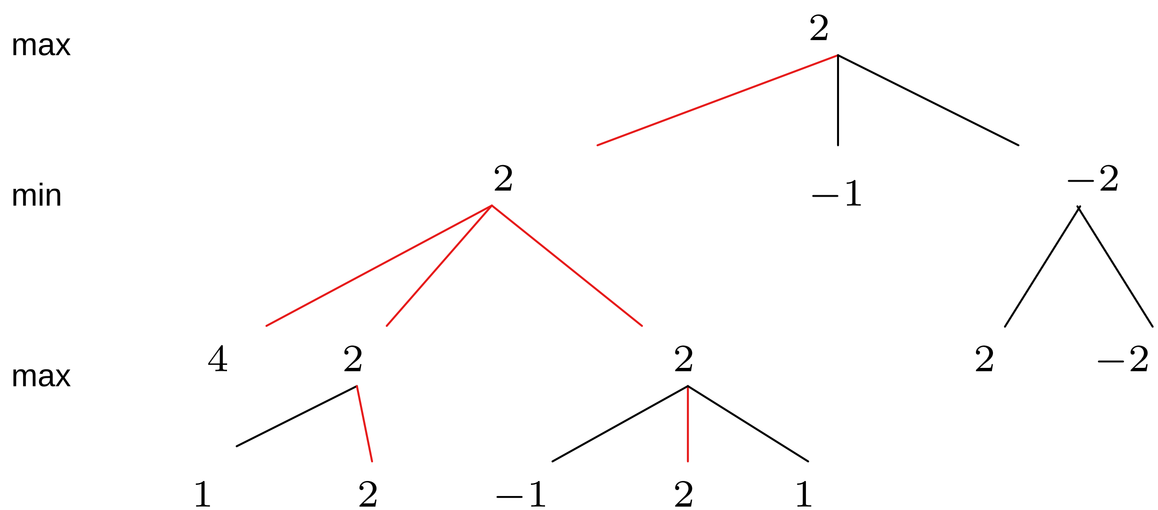
Exercise 20 (a): Minimax algorithm.



Minimax algorithm

Exercise 20 (a): Minimax algorithm.

Player 1

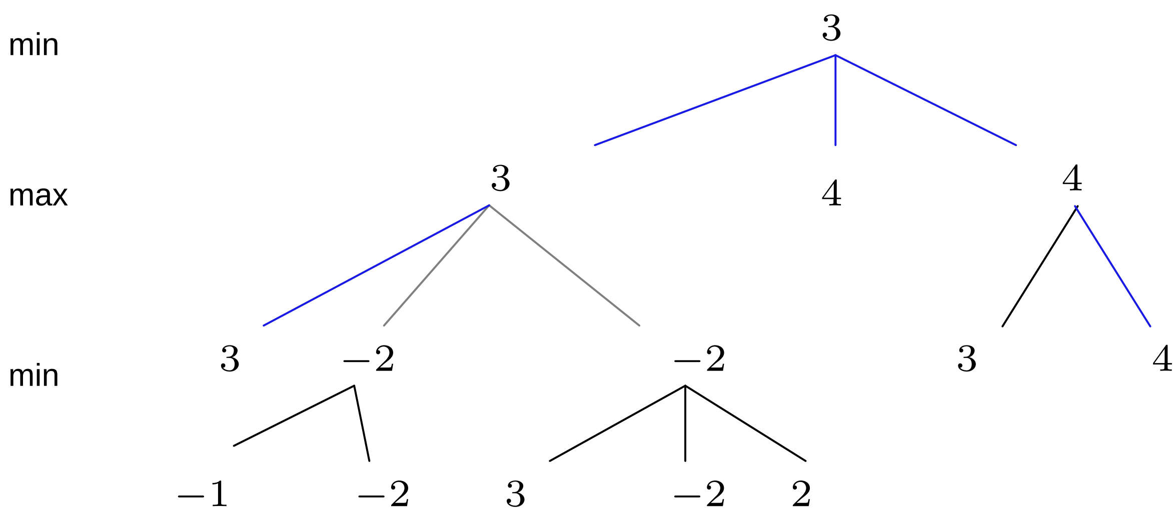


Value for Player 1: 2

Minimax algorithm

Exercise 20 (a): Minimax algorithm.

Player 2



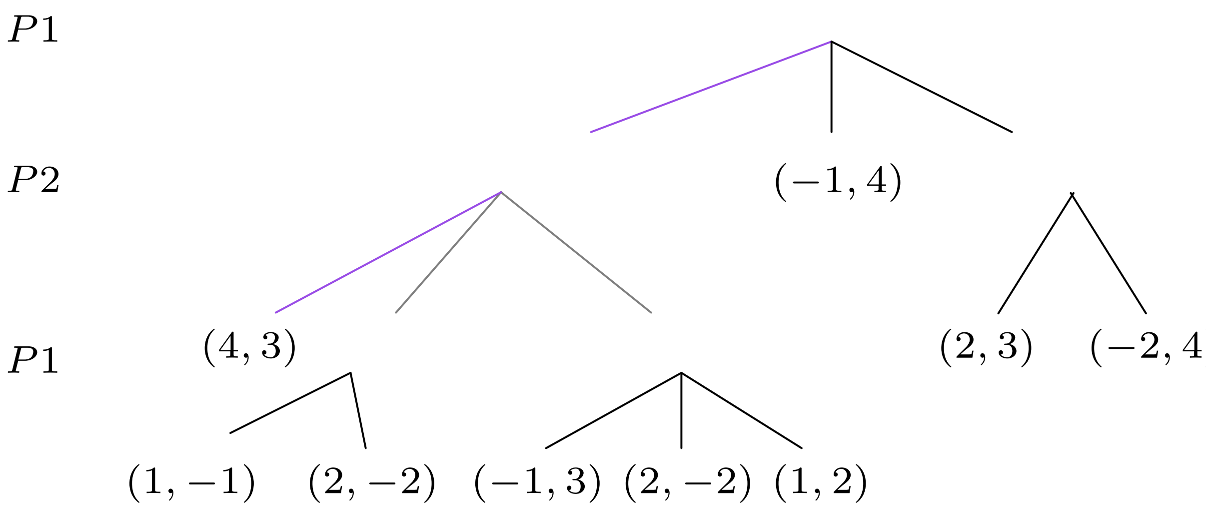
Value for Player 1: 2

Value for Player 2: 3

Minimax algorithm

Exercise 20 (a): Minimax algorithm.

Playing these strategies



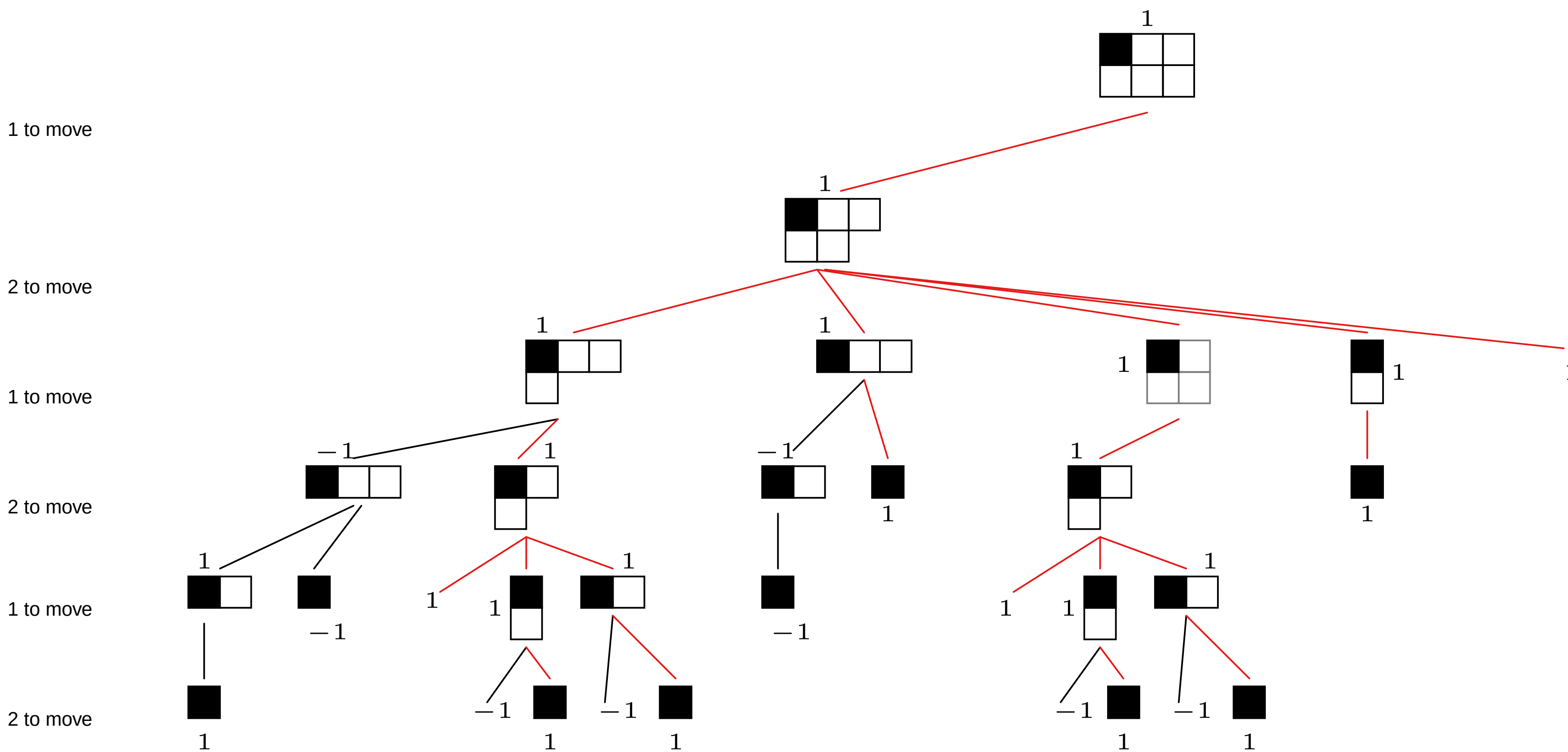
Value for Player 1: 2

Value for Player 2: 3

(2×3)-Chomp

Exercise 21 (a): Minimax algorithm.

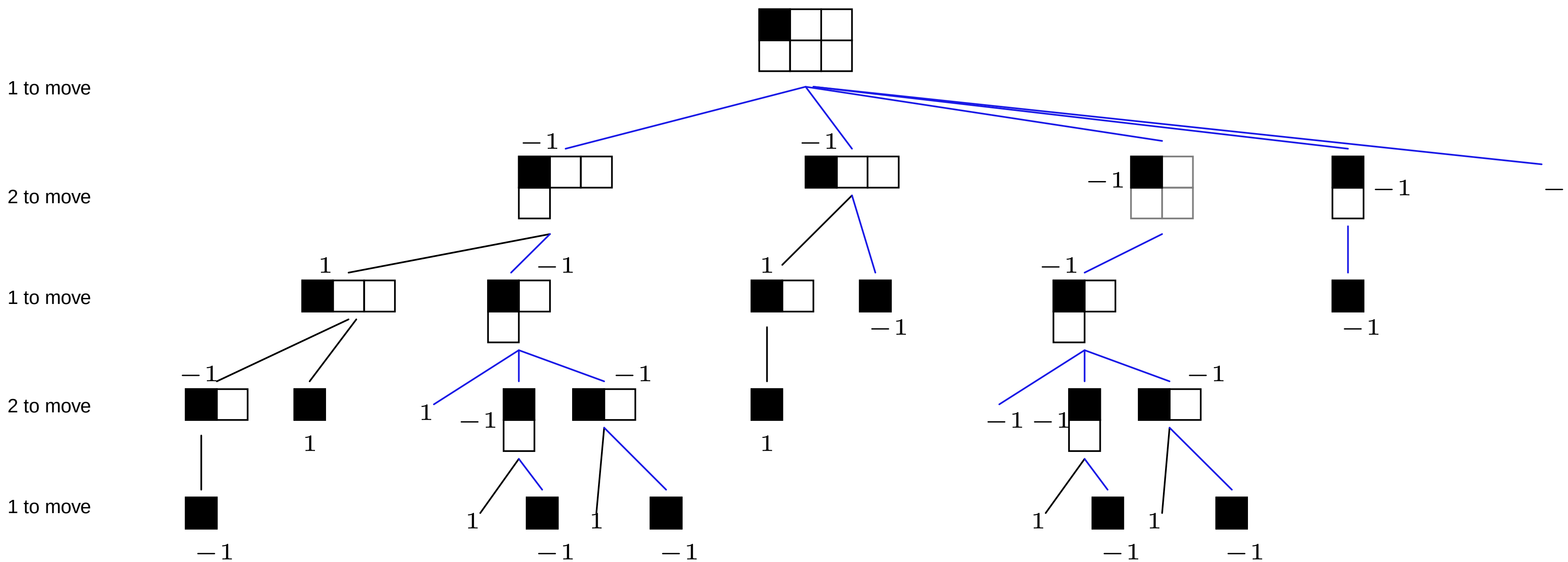
Alpha-beta pruning for Player 1 (Player 1 has winning strategy)



(2×3)-Chomp

Exercise 21 (a): Minimax algorithm.

Player 1 plays different opening move (then Player 2 can force a win)



Repeated Prisoner’s Dilemma

Exercise 22 (a): Repeated Prisoner’s Dilemma.

The hint says, for Player 2, to consider the strategy which

- defects five times in a row and
- in the last round
  - cooperates if the other side cooperated five times,
  - defects otherwise.

As an abbreviation, we call this strategy AAD, for ‘almost always defect’.

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

The hint says, for Player 2, to consider the strategy AAD, for 'almost always defect'.

Many plays against AAD leads to mutual defection for all six rounds, giving a pay-off of $6 \times (-8) = -48$ for both players.

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

The hint says, for Player 2, to consider the strategy AAD, for 'almost always defect'.

Many plays against AAD leads to mutual defection for all six rounds, giving a pay-off of $6 \times (-8) = -48$ for both players.

In order to do better the other strategy has to achieve a different outcome, and the only chance of that is to cooperate five times in a row and then defect. This leads to a pay-off for that strategy of $5 \times (-10) + 0 = -50$, which gives no improvement.

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

The hint says, for Player 2, to consider the strategy AAD, for 'almost always defect'.

Many plays against AAD leads to mutual defection for all six rounds, giving a pay-off of $6 \times (-8) = -48$ for both players.

In order to do better the other strategy has to achieve a different outcome, and the only chance of that is to cooperate five times in a row and then defect. This leads to a pay-off for that strategy of $5 \times (-10) + 0 = -50$, which gives no improvement.

This is almost enough to show that the following are equilibrium points:

$$(\text{AlwaysD}, \text{AAD}) \qquad (\text{AAD}, \text{AAD}).$$

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

This is almost enough to show that the following are equilibrium points:

$$(\textsc{AlwaysD}, \text{AAD}) \qquad\qquad (\text{AAD}, \text{AAD}).$$

So let's now show that these are indeed equilibrium points.

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

This is almost enough to show that the following are equilibrium points:

$$(\text{AlwaysD}, \text{AAD}) \qquad\qquad (\text{AAD}, \text{AAD}).$$

So let's now show that these are indeed equilibrium points.

If Player 1 changes his mind, then he will be worse off:

- We know that Player 2 will defect on the first five moves.
  - We know that cooperating five times to get him to cooperate in round 6 doesn't give a better pay-off.

  Hence our best answer leads to him defecting 6 times.
- The best play against defecting 6 times is also to defect 6 times. Both the given strategies do just that.

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

This is almost enough to show that the following are equilibrium points:

$$(\textsc{AlwaysD}, \text{AAD}) \qquad (\text{AAD}, \text{AAD}).$$

So let's now show that these are indeed equilibrium points.

If Player 1 changes his mind, then he will be worse off:

- We know that Player 2 will defect on the first five moves.
  - We know that cooperating five times to get him to cooperate in round 6 doesn't give a better pay-off.

  Hence our best answer leads to him defecting 6 times.
- The best play against defecting 6 times is also to defect 6 times. Both the given strategies do just that.

The argument for Player 2 is almost identical.

Repeated Prisoner’s Dilemma

Exercise 22 (a): Repeated Prisoner’s Dilemma.

This is almost enough to show that the following are equilibrium points:

$$(\textsc{AlwaysD}, \text{AAD}) \qquad\qquad (\text{AAD}, \text{AAD}).$$

$(\textsc{AlwaysD}, \text{AAD})$ and (AAD, AAD) are not subgame equilibrium points since there are subgames where they would cooperate (on the sixth round), but that is known not to be an equilibrium point strategy.

ALWAYSD is collectively stable

Exercise 23 (a): ALWAYSD **is collectively stable**

Show: No strategy can get higher pay-off against ALWAYSD than ALWAYSD against itself.

AlwaysD is collectively stable

Exercise 23 (a): AlwaysD **is collectively stable**

Show: No strategy can get higher pay-off against AlwaysD than AlwaysD against itself.

When AlwaysD plays against itself we get a game where both parties get pay-off P for each round.

ALWAYSD is collectively stable

Exercise 23 (a): ALWAYSD **is collectively stable**

Show: No strategy can get higher pay-off against ALWAYSD than ALWAYSD against itself.

When ALWAYSD plays against itself we get a game where both parties get pay-off P for each round.

But any strategy playing against ALWAYSD in any round can

- defect and get pay-off P or
- cooperate and get pay-off S.

ALWAYSD is collectively stable

Exercise 23 (a): ALWAYSD **is collectively stable**

Show: No strategy can get higher pay-off against ALWAYSD than ALWAYSD against itself.

When ALWAYSD plays against itself we get a game where both parties get pay-off P for each round.

But any strategy playing against ALWAYSD in any round can

- defect and get pay-off P or
- cooperate and get pay-off S.

But $P \geq S$, and so (as we have known all along) the best answer against somebody who is known always to defect is to defect in turn.

ALWAYSD is collectively stable

Exercise 23 (a): ALWAYSD **is collectively stable**

Show: No strategy can get higher pay-off against ALWAYSD than ALWAYSD against itself.

When ALWAYSD plays against itself we get a game where both parties get pay-off P for each round.

But any strategy playing against ALWAYSD in any round can

- defect and get pay-off P or
- cooperate and get pay-off S.

But $P \geq S$, and so (as we have known all along) the best answer against somebody who is known always to defect is to defect in turn.

In other words, there is nothing the other strategy can do which would allow it to achieve a higher pay-off against ALWAYSD than ALWAYSD does against itself.

The Hawk-Dove Game

Exercise 24 (a): The Hawk-Dove Game

The matrix for the game is

$$\begin{vmatrix} -40 & 40 \\ 0 & 10, \end{vmatrix}$$

where the pay-offs are given for the row player and the symmetry of the game means that they we can read off pay-offs for both, Player 1 and Player 2 from this.

The Hawk-Dove Game

Exercise 24 (a): The Hawk-Dove Game

The matrix for the game is

$$\begin{vmatrix} -40 & 40 \\ 0 & 10, \end{vmatrix}$$

where the pay-offs are given for the row player.

A population with a proportion p of HAWKS and $1 - p$ DOVES is stable iff the pay-off per round for a HAWK is the same as that for a DOVE.

The Hawk-Dove Game

Exercise 24 (a): The Hawk-Dove Game

The matrix for the game is

$$\begin{vmatrix} -40 & 40 \\ 0 & 10, \end{vmatrix}$$

where the pay-offs are given for the row player.

A population with a proportion p of HAWKS and $1-p$ DOVES is stable iff the pay-off per round for a HAWK is the same as that for a DOVE.

The former is

$$-40p + 40(1-p) = 40 - 80p,$$

while the latter is

$$10(1-p) = 10 - 10p.$$

The Hawk-Dove Game

Exercise 24 (a): The Hawk-Dove Game

The matrix for the game is

$$\begin{vmatrix} -40 & 40 \\ 0 & 10, \end{vmatrix}$$

where the pay-offs are given for the row player.

A population with a proportion p of HAWKS and $1-p$ DOVES is stable iff the pay-off per round for a HAWK is the same as that for a DOVE.

The former is

$$-40p + 40(1-p) = 40 - 80p,$$

while the latter is

$$10(1-p) = 10 - 10p.$$

The two are equal iff

$$40 - 80p = 10 - 10p, \qquad \text{that is iff} \qquad p = 3/7.$$