CS3192 Section 1

Slides for some solutions

Andrea Schalk

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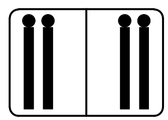
$(2,2)$-Nim

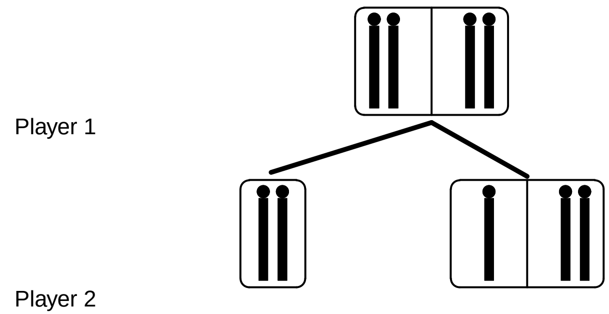
Exercise 1 (a): $(2,2)$**-Nim.**

P.layer 1



$(2,2)$-Nim

Exercise 1 (a): $(2,2)$-Nim.



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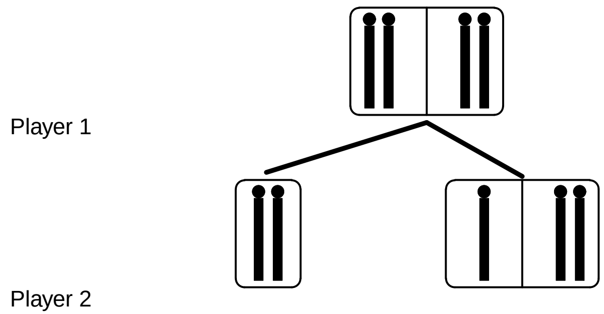

We are taking symmetry into account here.

$(2, 2)$-Nim

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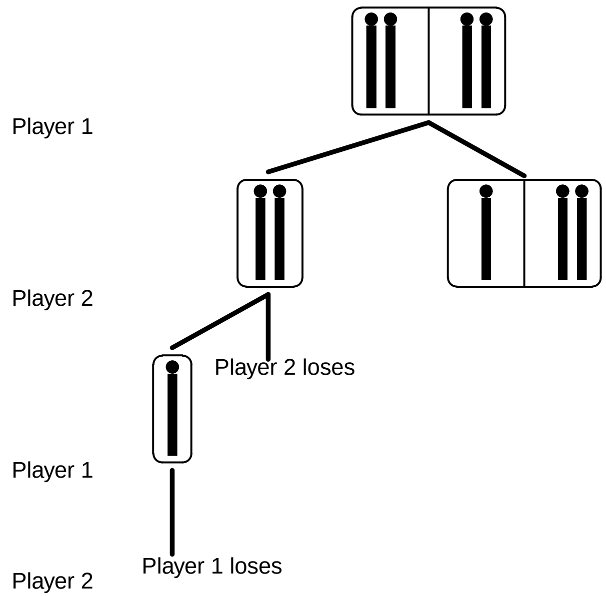

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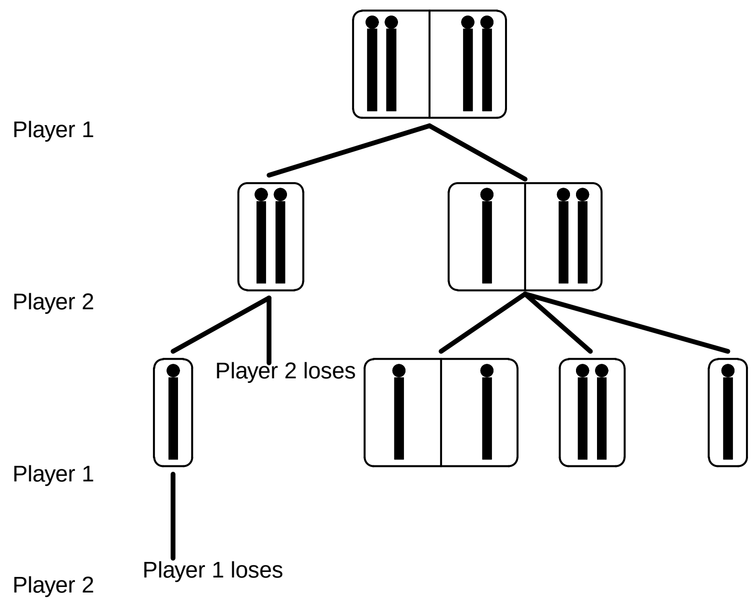

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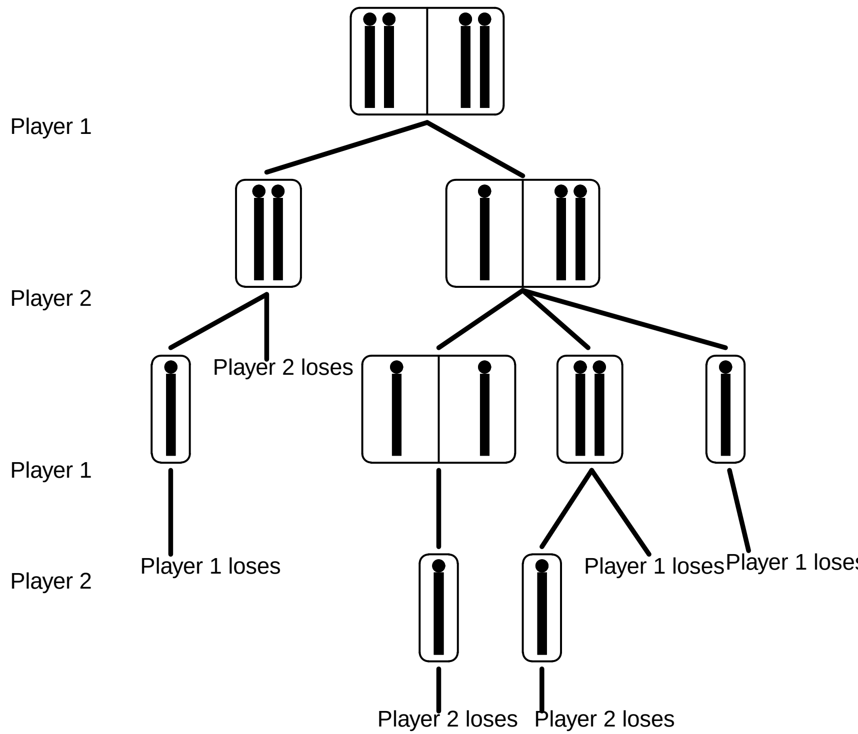
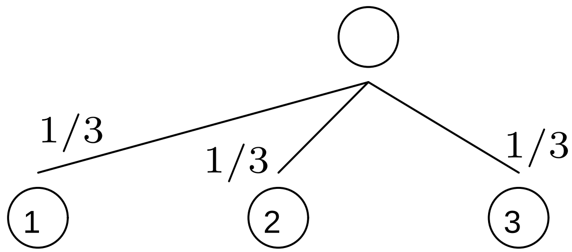


Throwing dice

Exercise 2 (a): Throwing two 3-faced dice.

First throw of one die:



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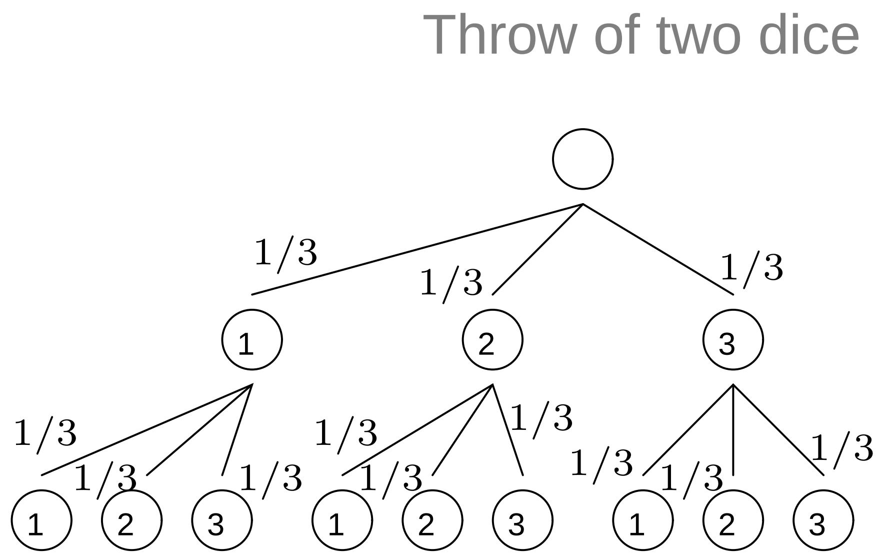
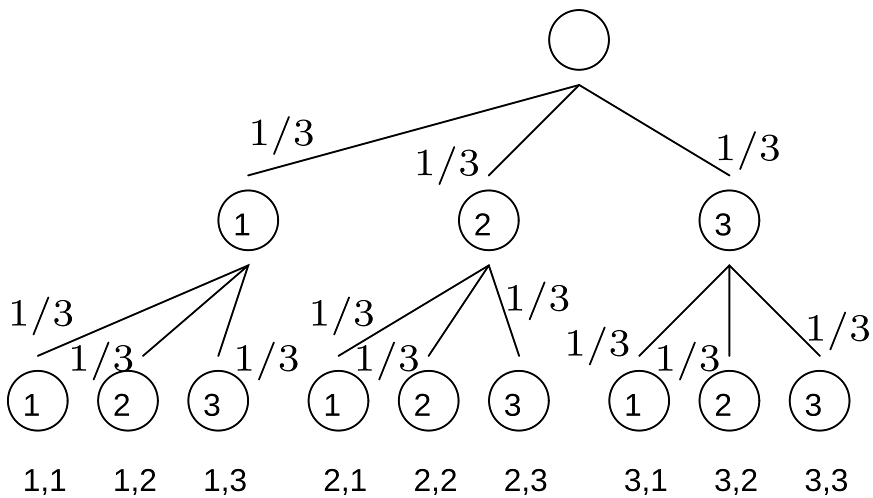


Throwing dice

Exercise 2 (a): Throwing two 3-faced dice.

With outcomes



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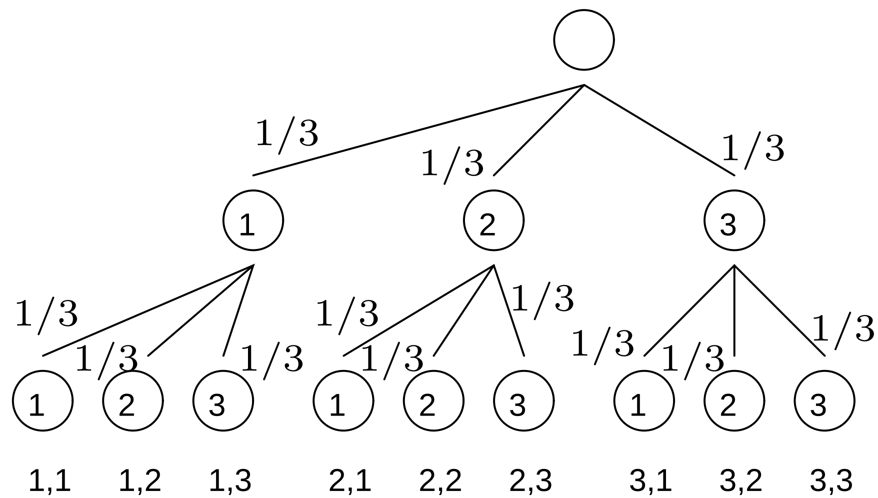

The probability for any of the leaves to occur is

$$1/3 \times 1/3 = 1/9.$$

Throwing dice

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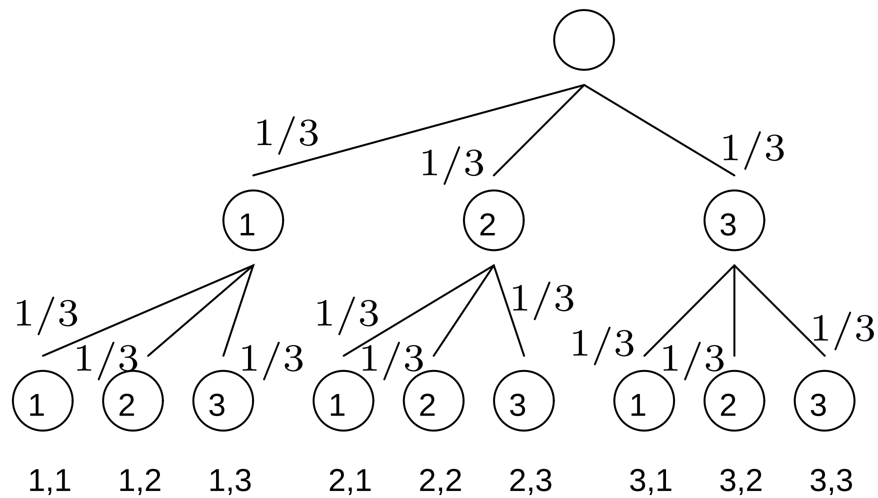


The probability for any of the leaves to occur is

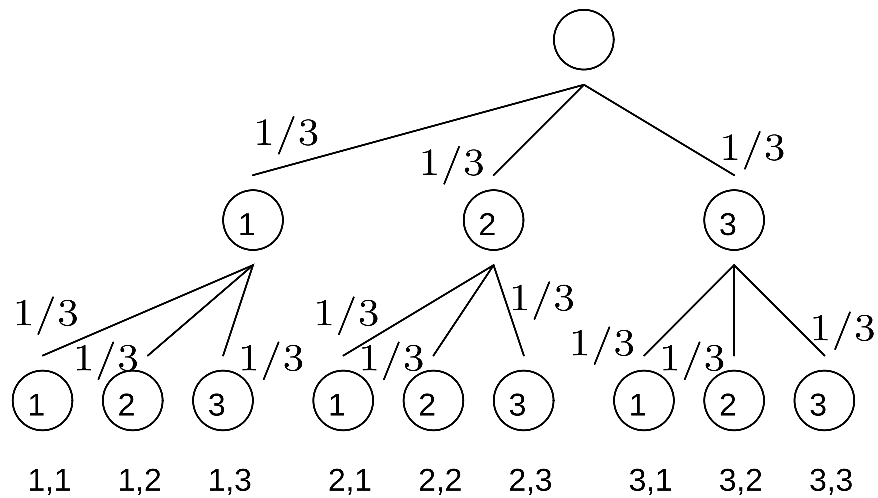
$$1/3 \times 1/3 = 1/9.$$

There is one leaf each for outcomes of throwing two 1s, two 2s or two 3s, but two leaves each for the outcome where the two numbers shown are different.

Throwing dice

Exercise 2 (a): **Throwing two 3-faced dice.**

With outcomes



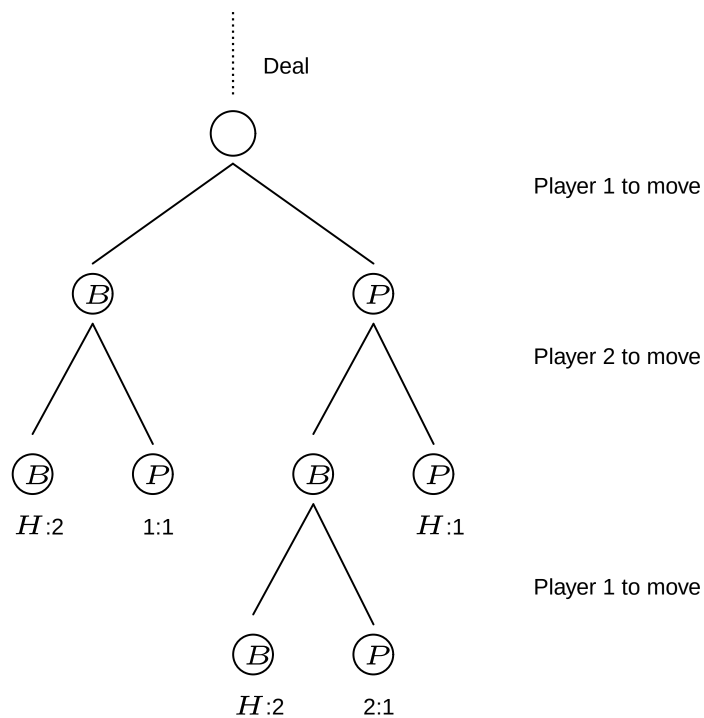
Hence these are the probabilities for the outcomes:

1,1	1,2	1,3	2,2	2,3	3,3
$1/9$	$2/9$	$2/9$	$1/9$	$2/9$	$1/9$

Simplified Poker

Exercise 3 (a): Simplified Poker

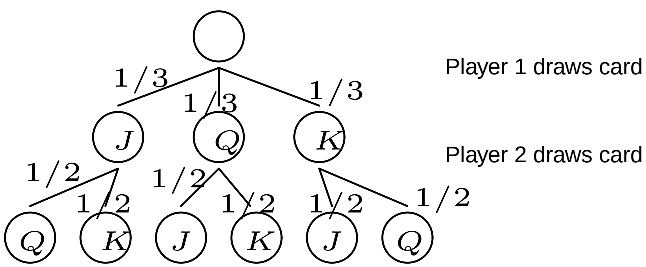
Here is the game tree when ignoring the deal.



Simplified Poker

Exercise 3 (a): Simplified Poker

The deal:

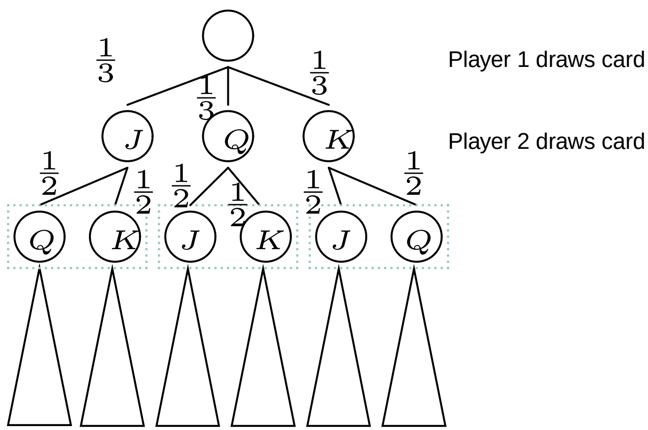


There are six possible deals: Player 1 might get any of the three cards, and Player 2 might then get any of the two remaining cards. Since all the possible combinations can occur, we have to multiply the possibilities to get $3 \times 2 = 6$ deals.

Simplified Poker

Exercise 3 (a): Simplified Poker

The whole tree:



There are six possible deals: Player 1 might get any of the three cards, and Player 2 might then get any of the two remaining cards. Since all the possible combinations can occur, we have to multiply the possibilities to get $3 \times 2 = 6$ deals.

Hence there will be 6 copies of the tree given here in the actual game tree. Furthermore, it will be necessary to indicate the information sets, that is the positions the player whose turn it is cannot distinguish between.

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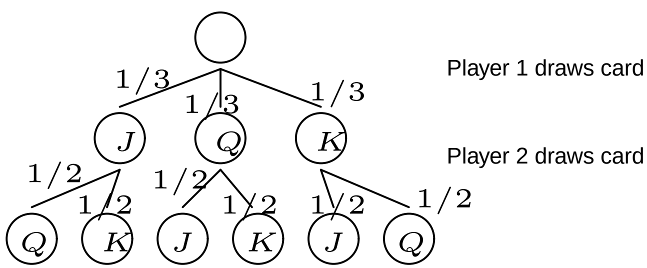

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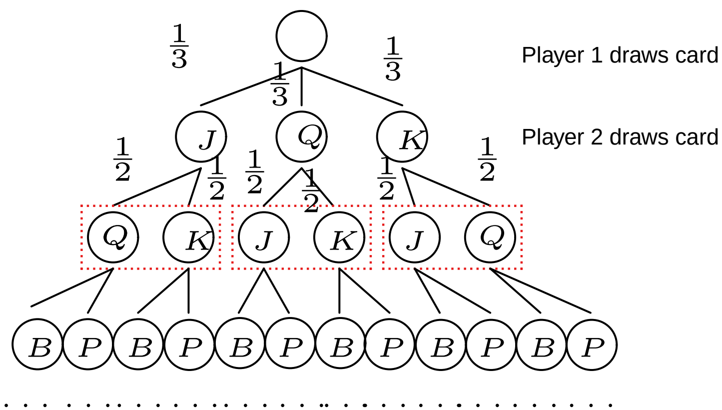

Hence there will be 6 copies of the tree given here in the actual game tree. Furthermore, it will be necessary to indicate the information sets, that is the positions the player whose turn it is cannot distinguish between.

For Player 1 this is fairly easy, see the picture on the left. However, the next move is Player 2's, and there the two nodes which belong to each information set are much further apart!

Simplified Poker

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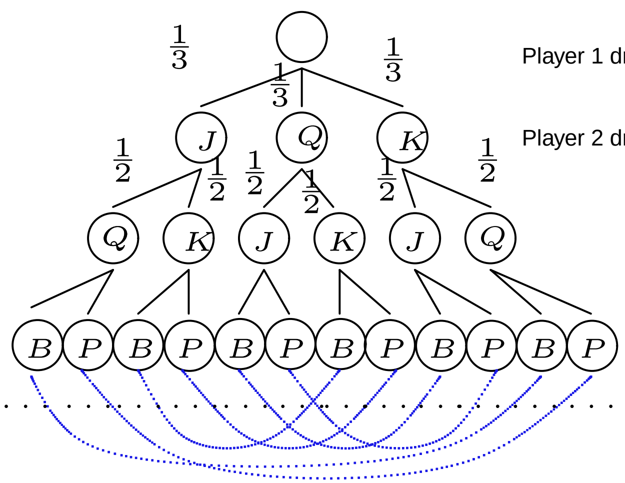


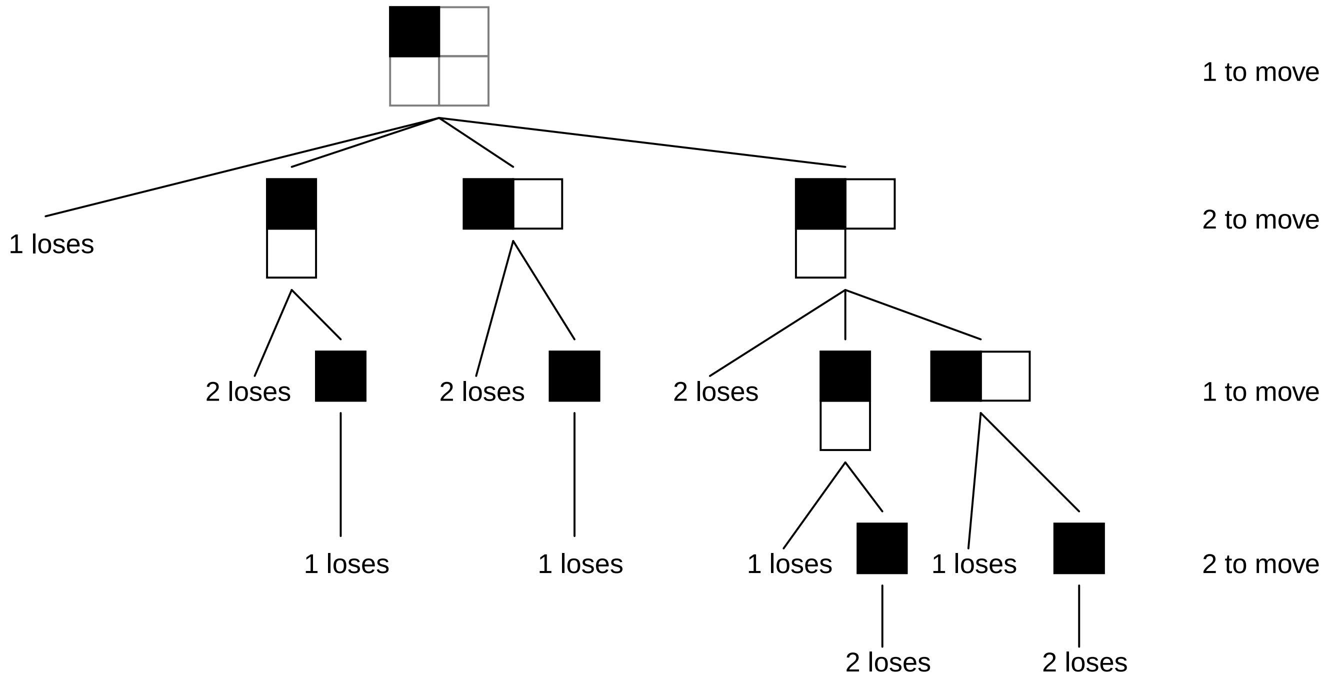
For Player 1 this is fairly easy, see the picture on the left. However, the next move is Player 2's, and there the two nodes which belong to each information set are much further apart!

For Player 2, two nodes are in the same information set if

- Player 2 has the same card in each node and
- Player 1's previous action (pass or bet) is the same.

(2×2)-Chomp

Exercise 4 (a): (2×2)**-Chomp,** number of strategies for Player 2.



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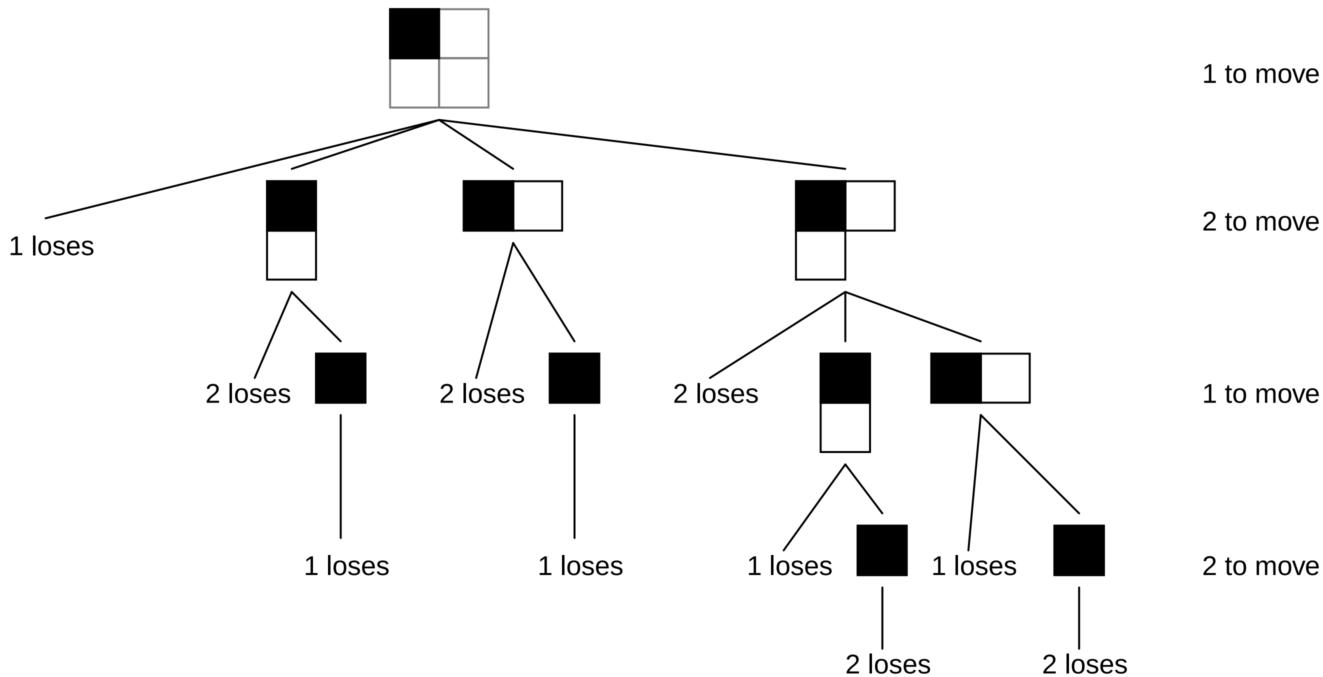

We use the recursive algorithm to count the strategies for Player 2. First of all, we fill in a '1' at every leaf—a place where there is obviously precisely one strategy for Player 2.

(2×2)-Chomp

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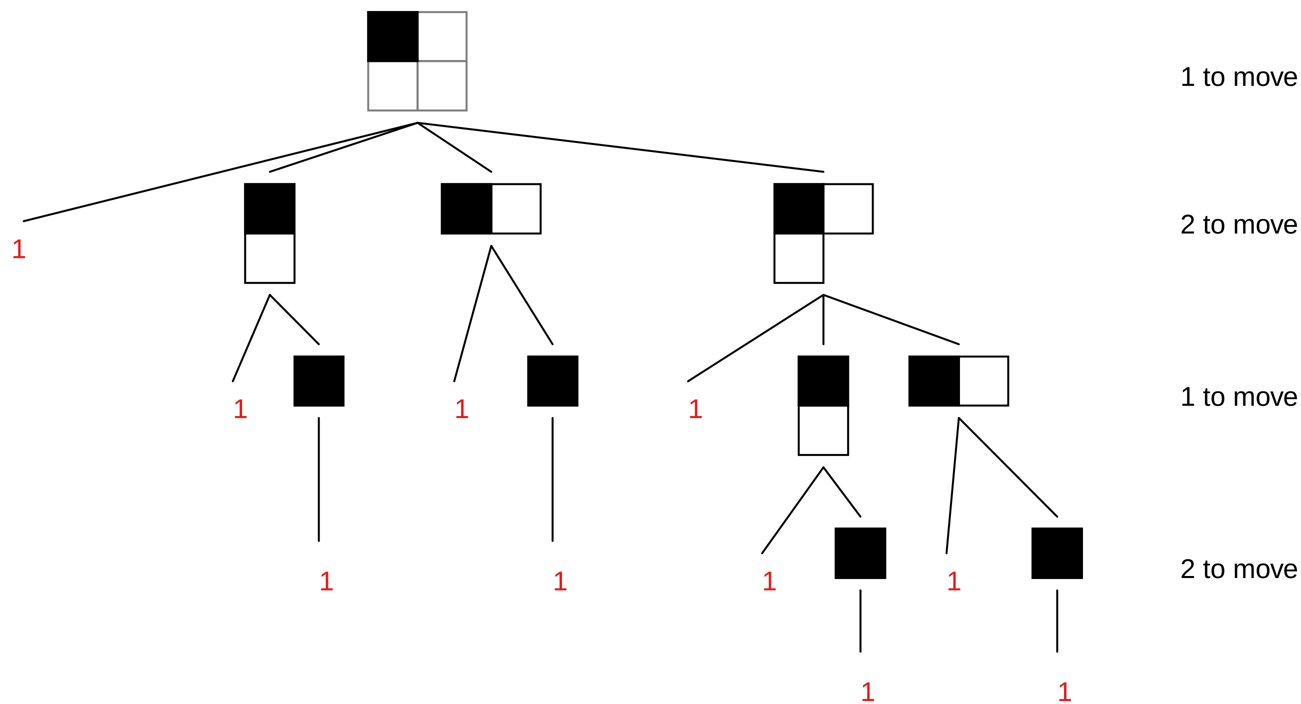

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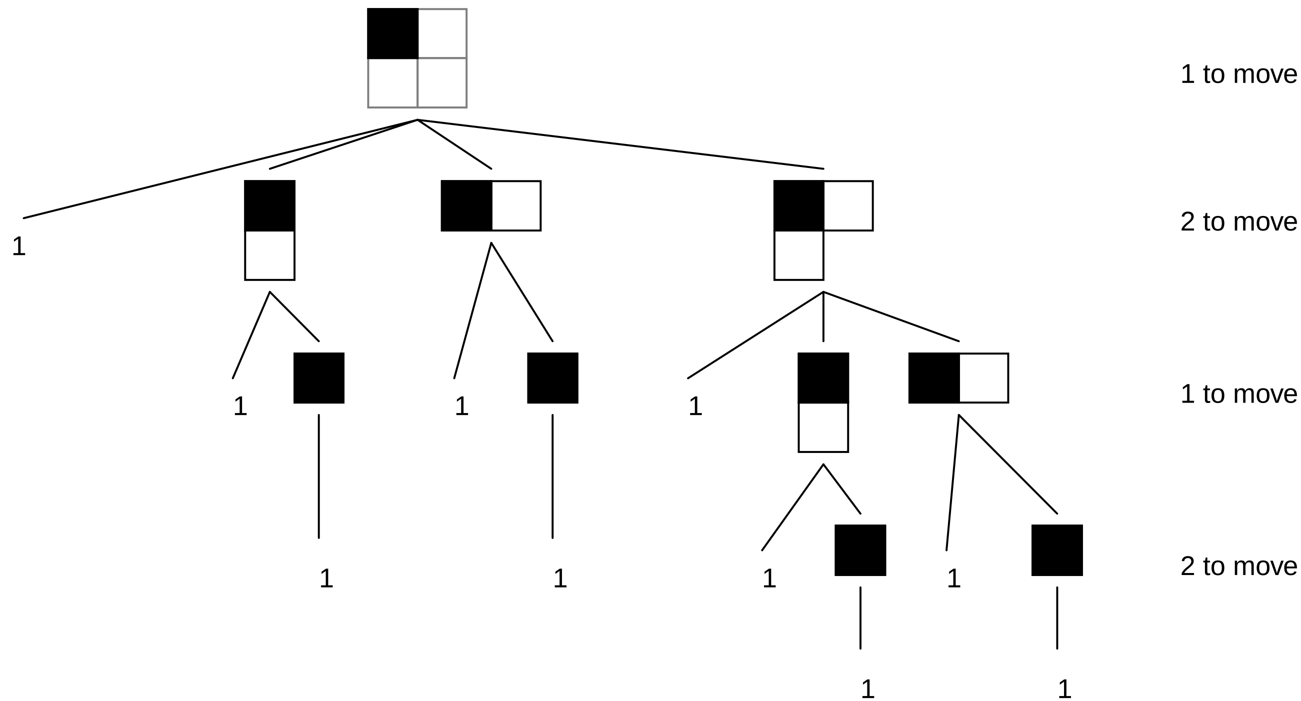
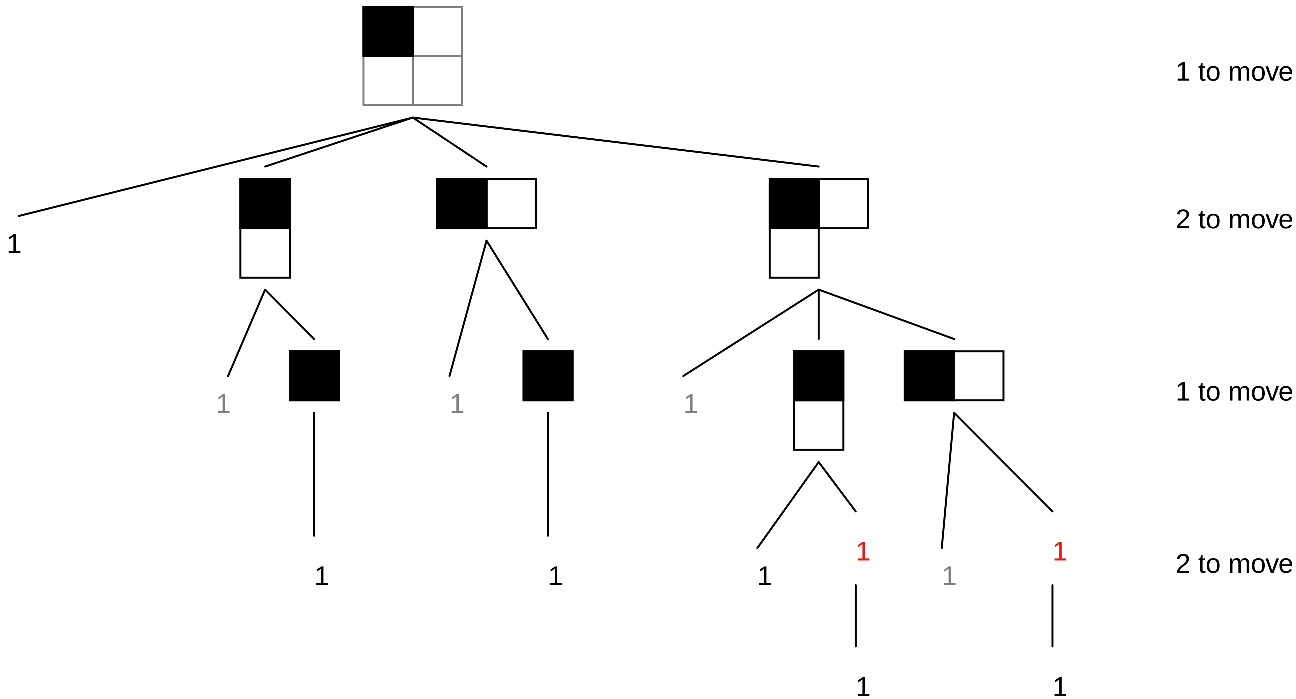


Now we fill in the number of strategies on the level above. This is a level where Player 2 makes the decision, so we have to add the numbers.

(2×2)-Chomp

Exercise 4 (a): (2×2)**-Chomp,** number of strategies for Player 2.



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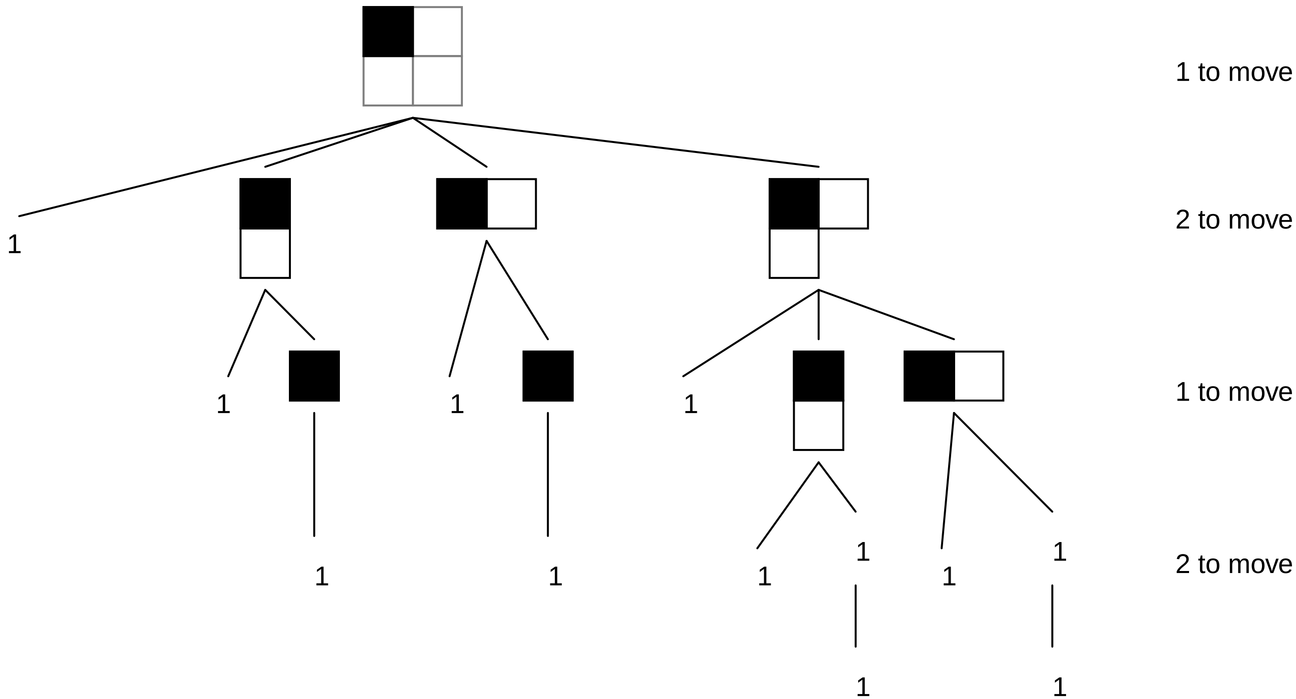
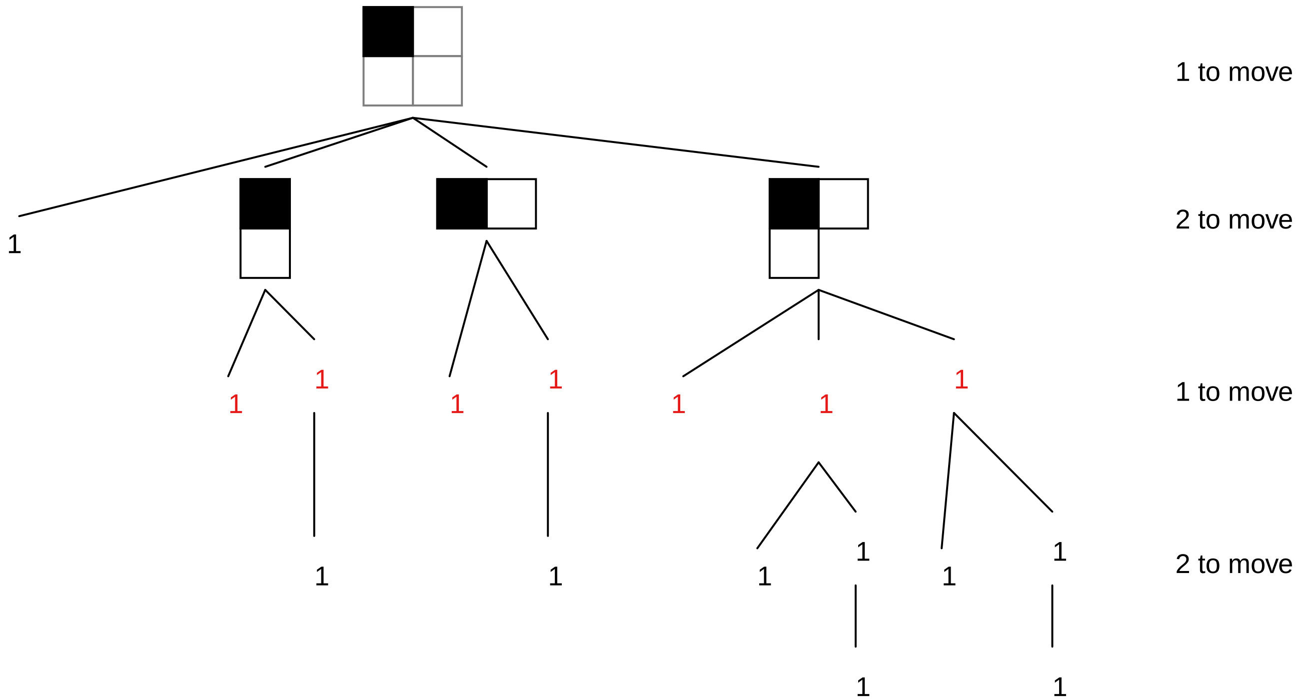


The next level up it is Player 1's turn, so we have to multiply numbers.

(2×2)-Chomp

Exercise 4 (a): (2×2)**-Chomp,** number of strategies for Player 2.



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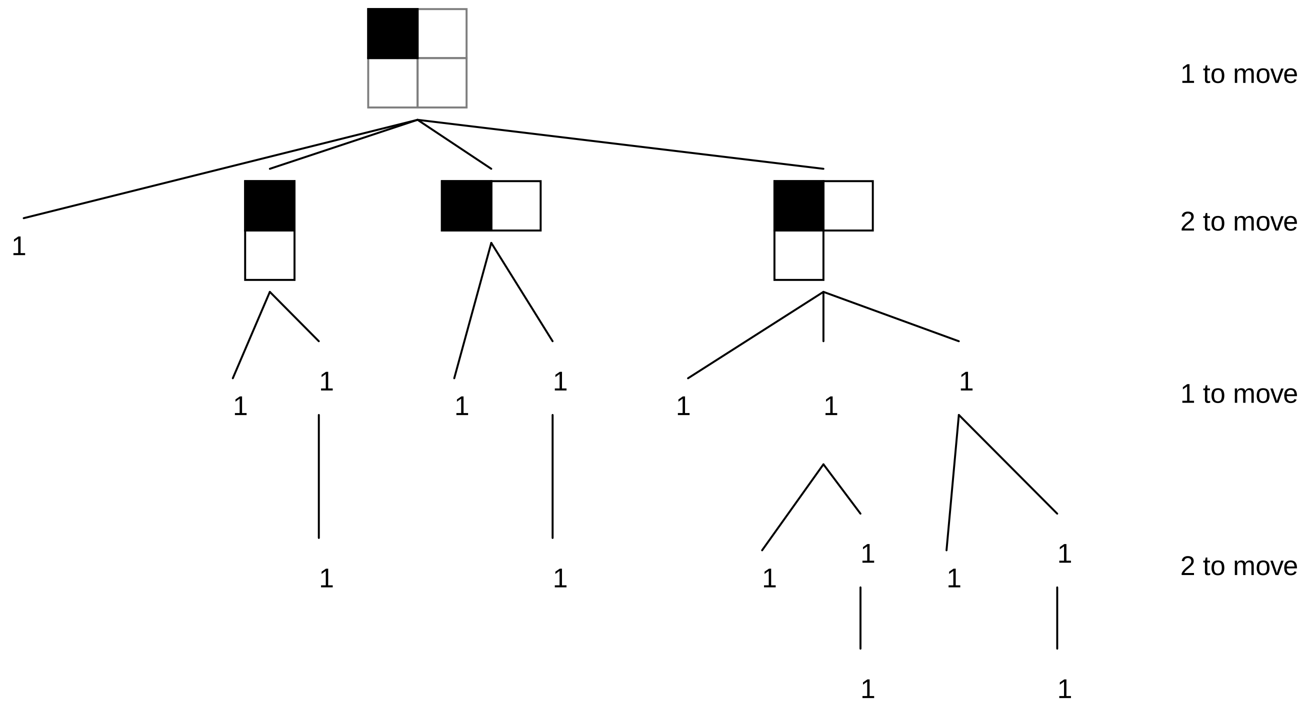
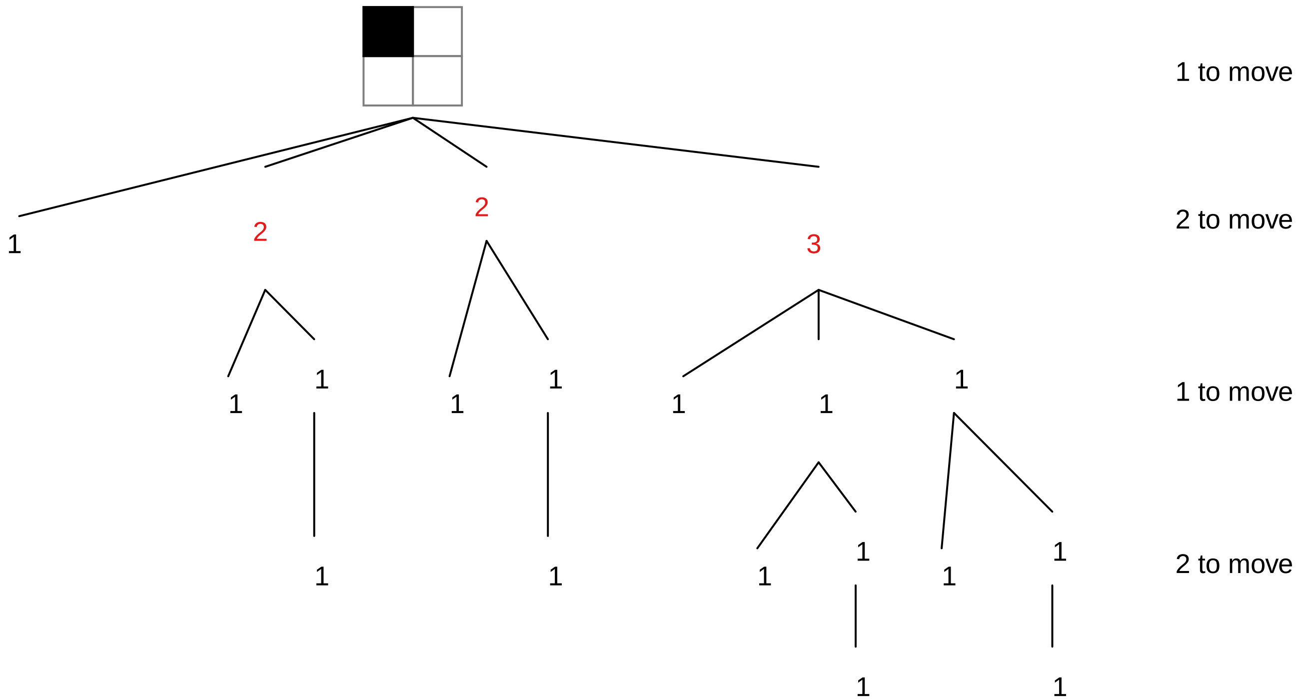


The next level up it's Player 2's turn again, so we have to add up numbers once more.

(2×2)-Chomp

Exercise 4 (a): (2×2)**-Chomp,** number of strategies for Player 2.



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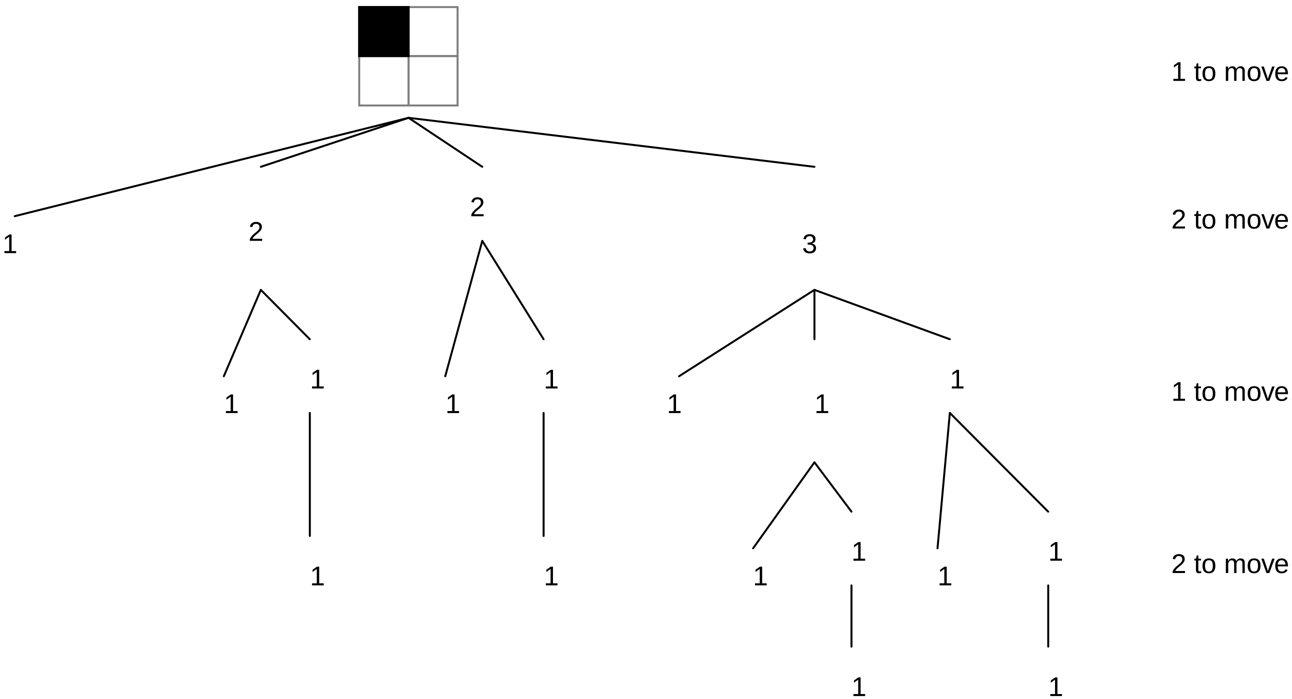
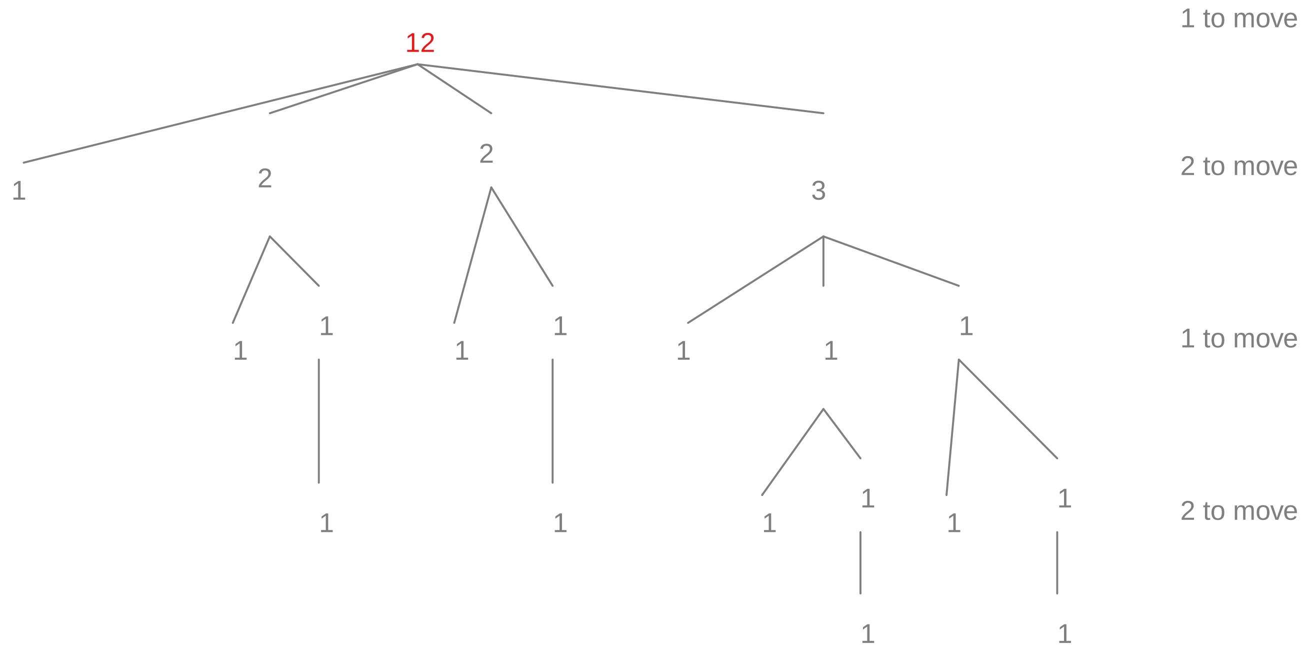


Lastly, Player 1 starts so we have to multiply the numbers from the level below.

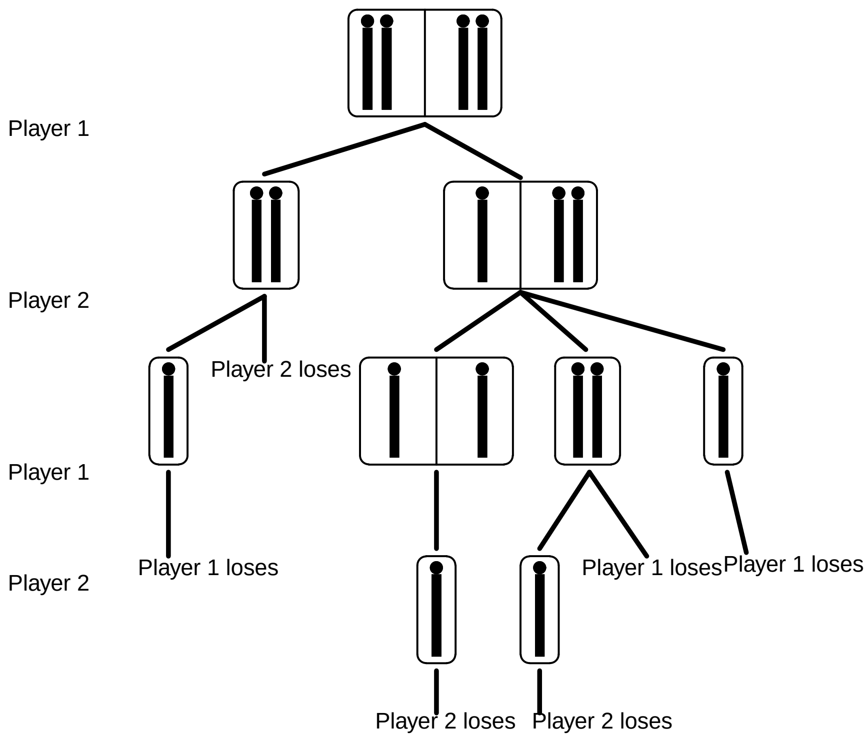
(2×2)-Chomp

Exercise 4 (a): (2×2)**-Chomp,** number of strategies for Player 2.



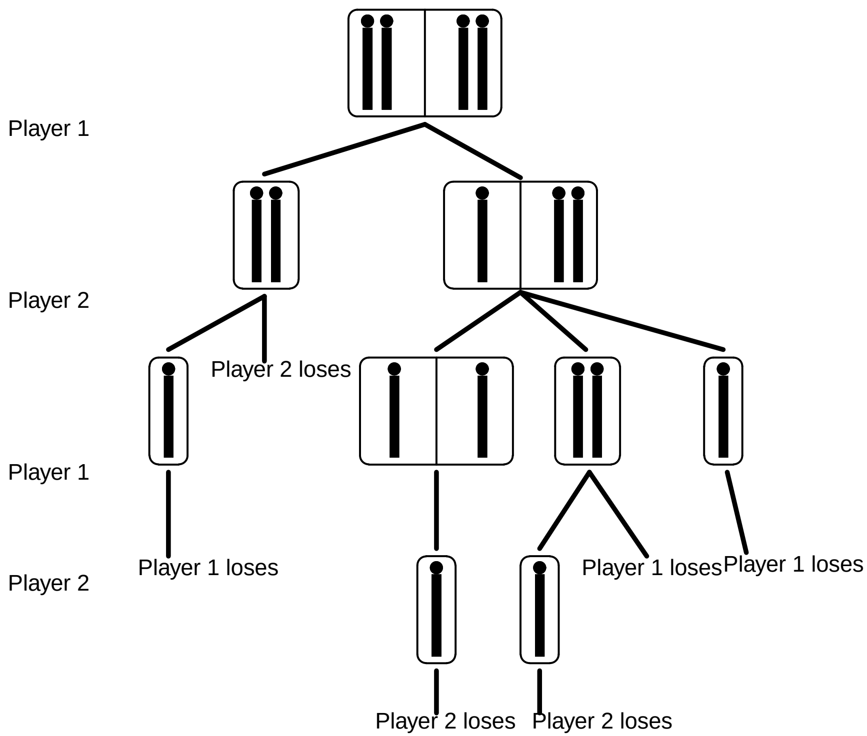
$(2,2)$-Nim

Exercise 5 (a): $(2,2)$**-Nim**, all strategies for both players.



$(2, 2)$-Nim

Exercise 5 (a): $(2, 2)$**-Nim**, all strategies for both players.



We use the same algorithm as for the previous exercise, but count a bit faster now.

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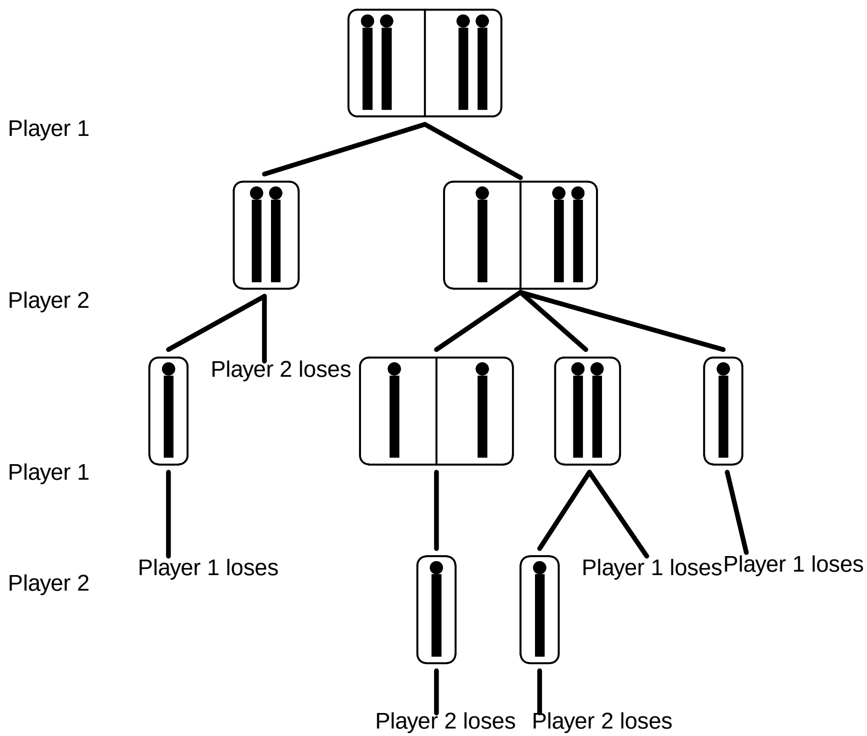

Player 1. We fill in 1s everywhere it is justified.

$(2,2)$-Nim

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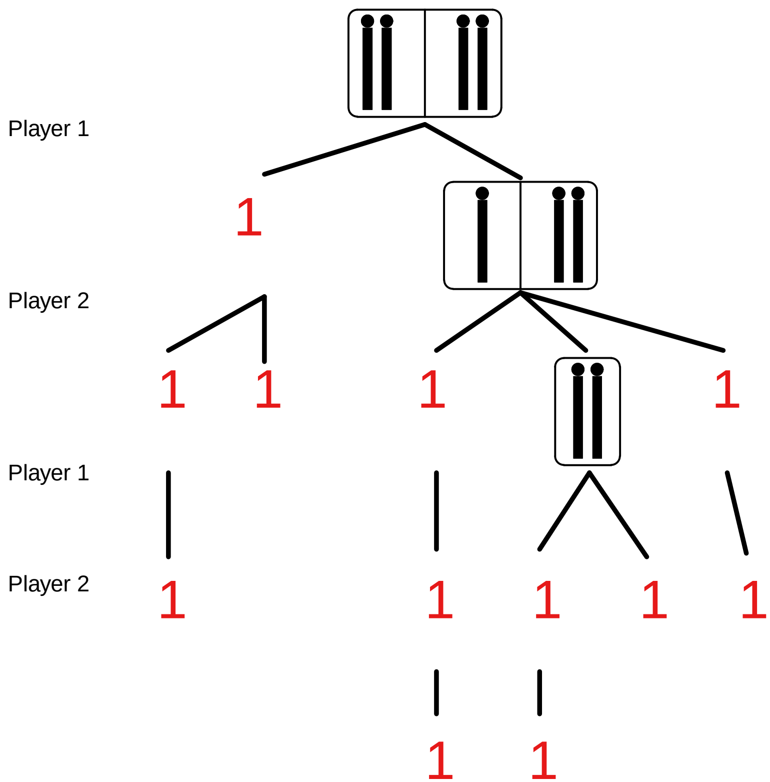
Player 1.



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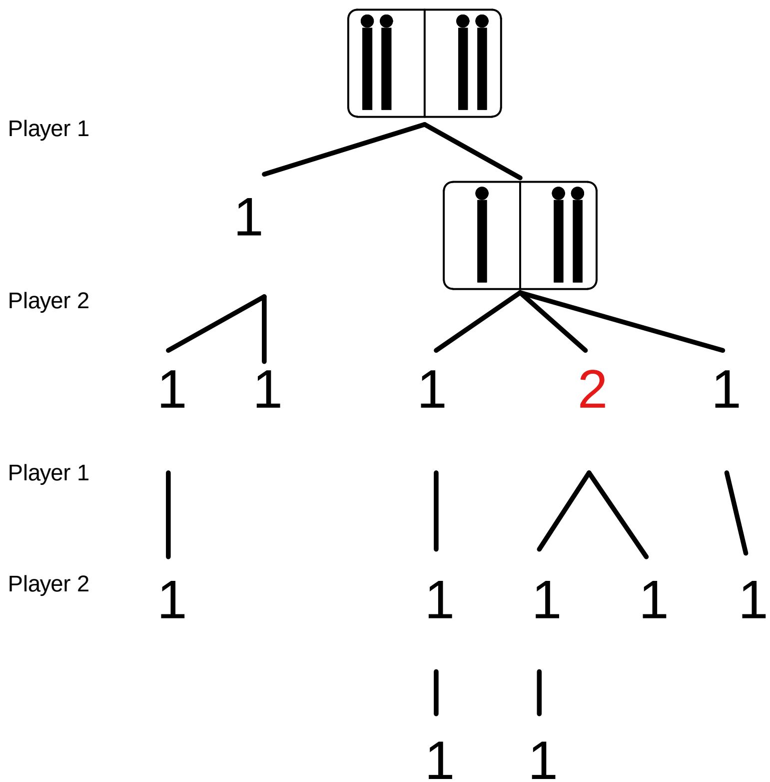

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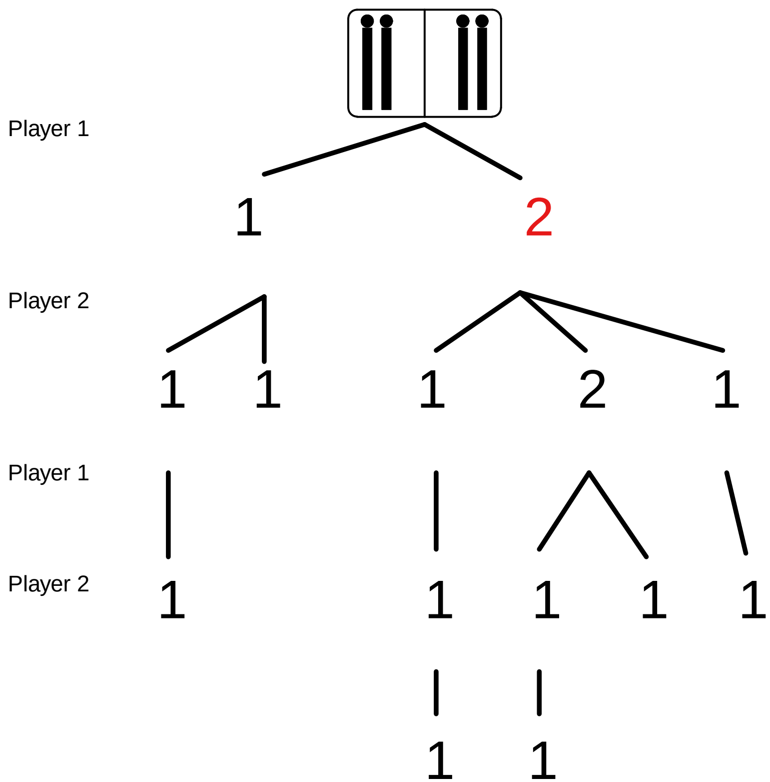

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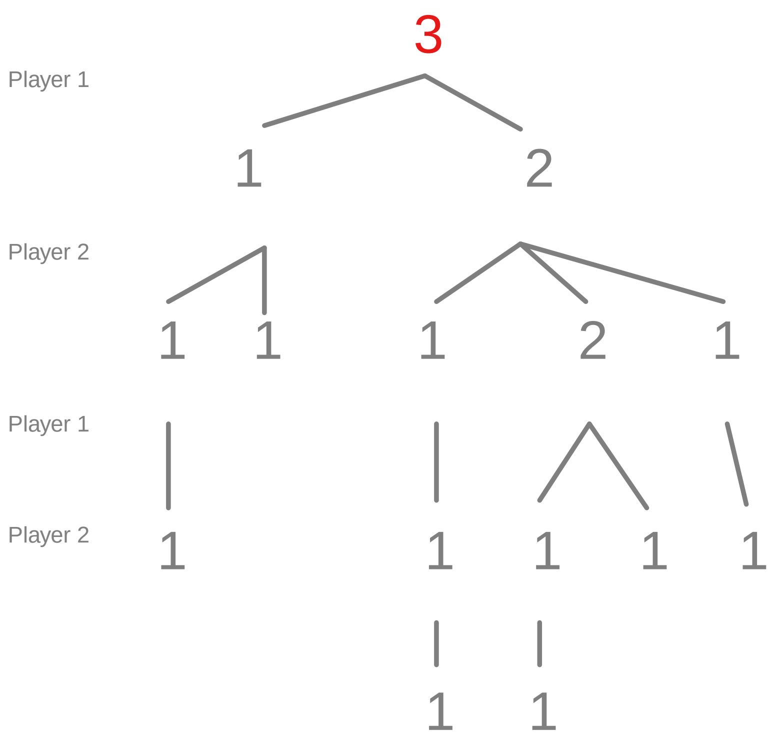


Player 1.

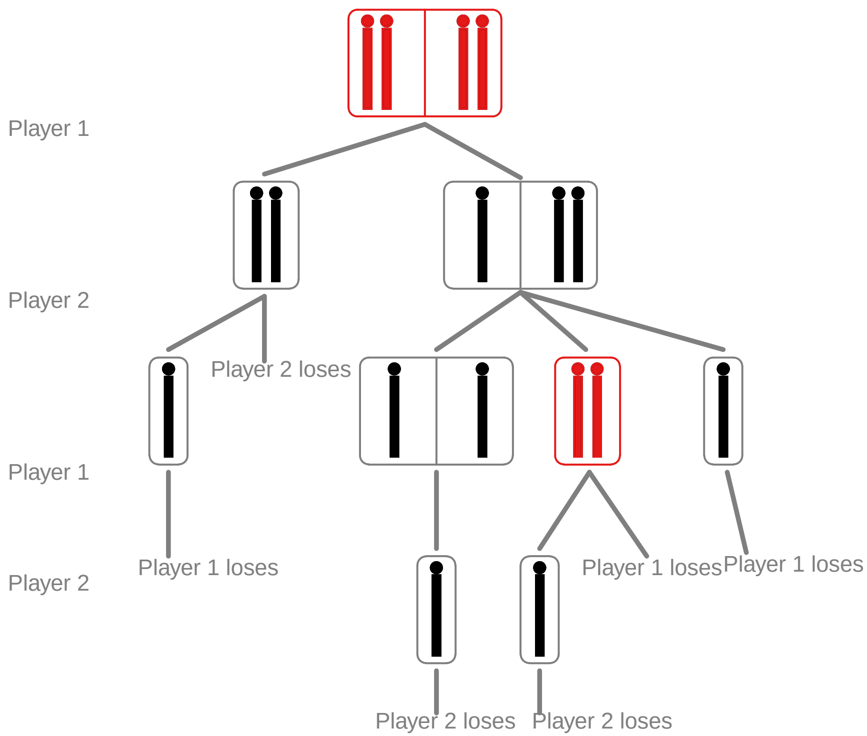
$(2, 2)$-Nim

Exercise 5 (a): $(2, 2)$**-Nim**, all strategies for both players.

Player 1. There's a faster way of trying to get a grip on this for very small games. Player 1 has two decision points.

$(2, 2)$-Nim

Exercise 5 (a): $(2, 2)$**-Nim**, all strategies for both players.



Player 1. Player 1 has two decision points.

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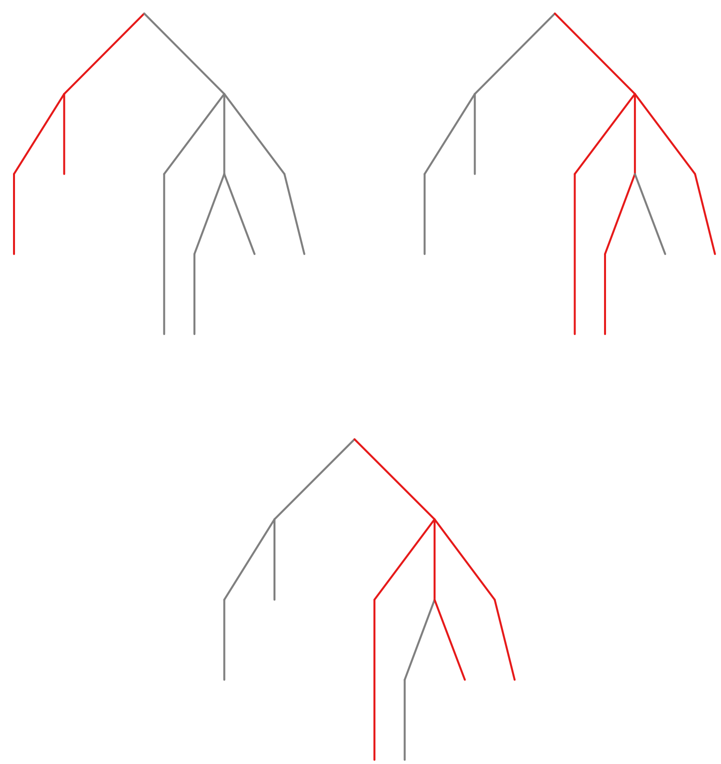
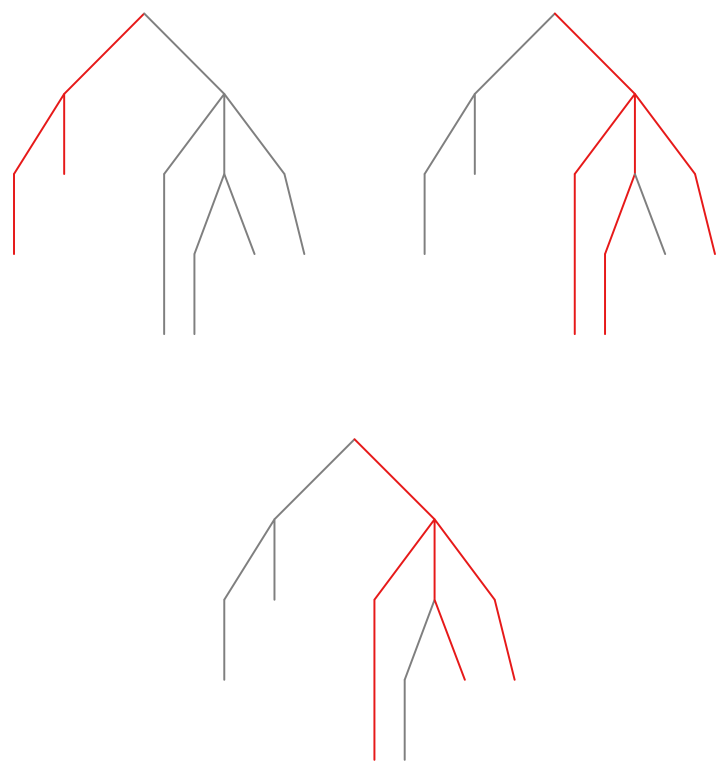


Player 1. Player 1 has two decision points. There are three strategies:

- 'Choose the left branch',
- 'choose the right branch and then the left branch at the next decision point' and
- 'choose the right branch and then the right branch again at the next decision point'.

$(2,2)$-Nim

Exercise 5 (a): $(2,2)$-Nim, all strategies for both players.



Player 1. There are three strategies:

- ‘Choose the left branch’,
- ‘choose the right branch and then the left branch at the next decision point’ and
- ‘choose the right branch and then the right branch again at the next decision point’.

Sensible names for these: 2 (for ‘choose the left branch, that is, take 2 matches from either pile’), $(1,1)$ and $(1,2)$.

$(2,2)$-Nim

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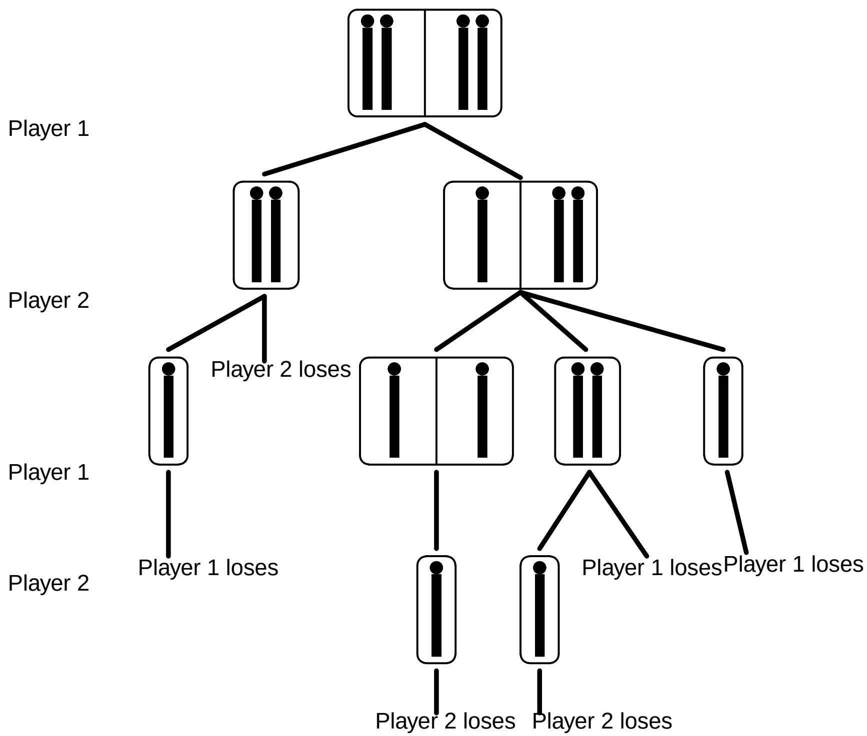

Player 2. Again, we fill in 1s where possible.

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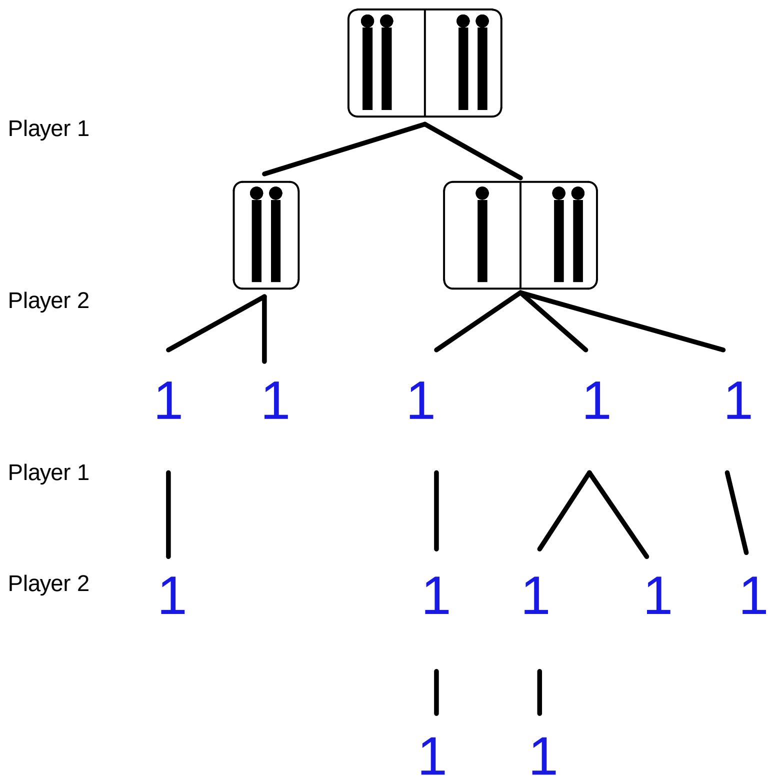

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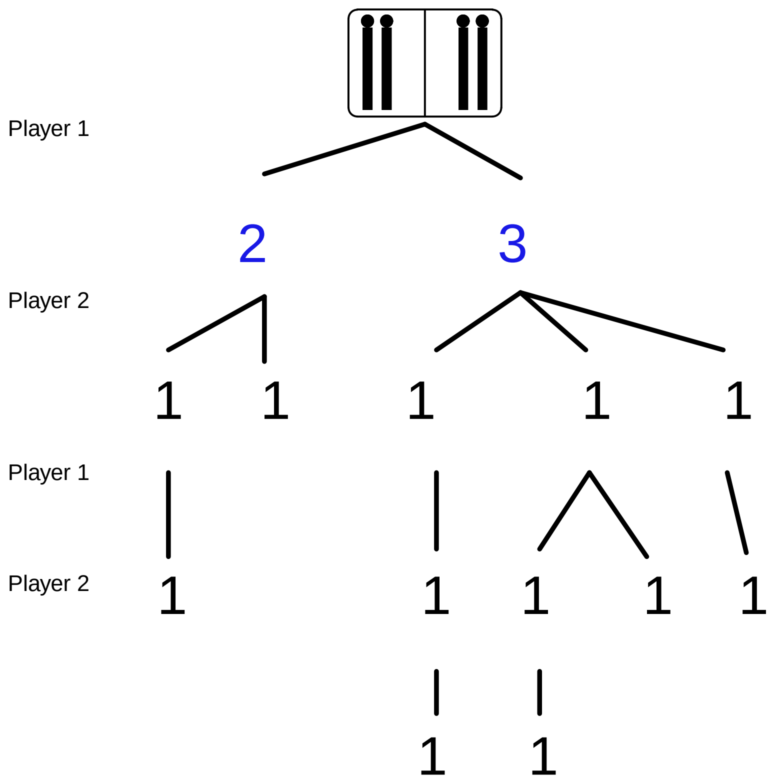

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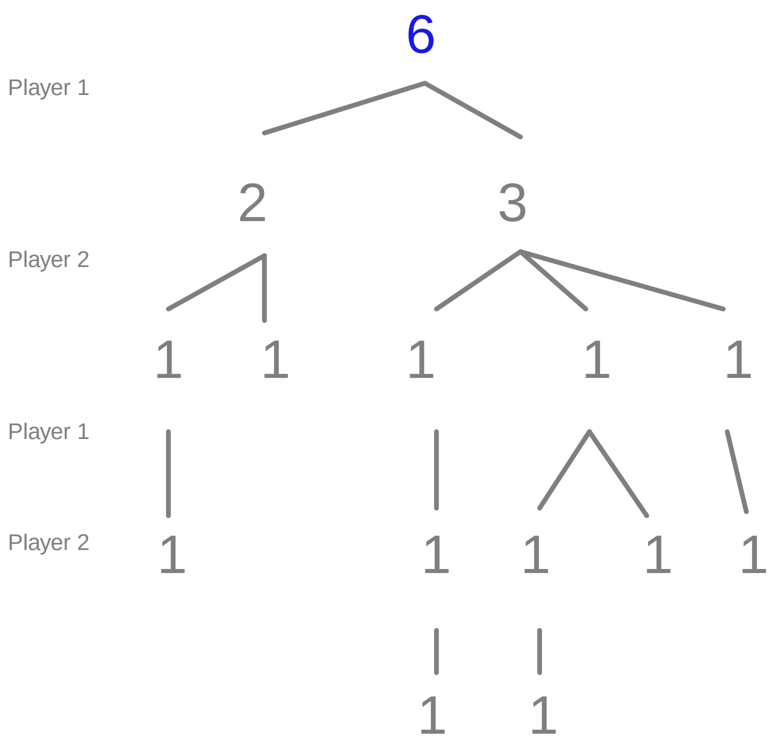

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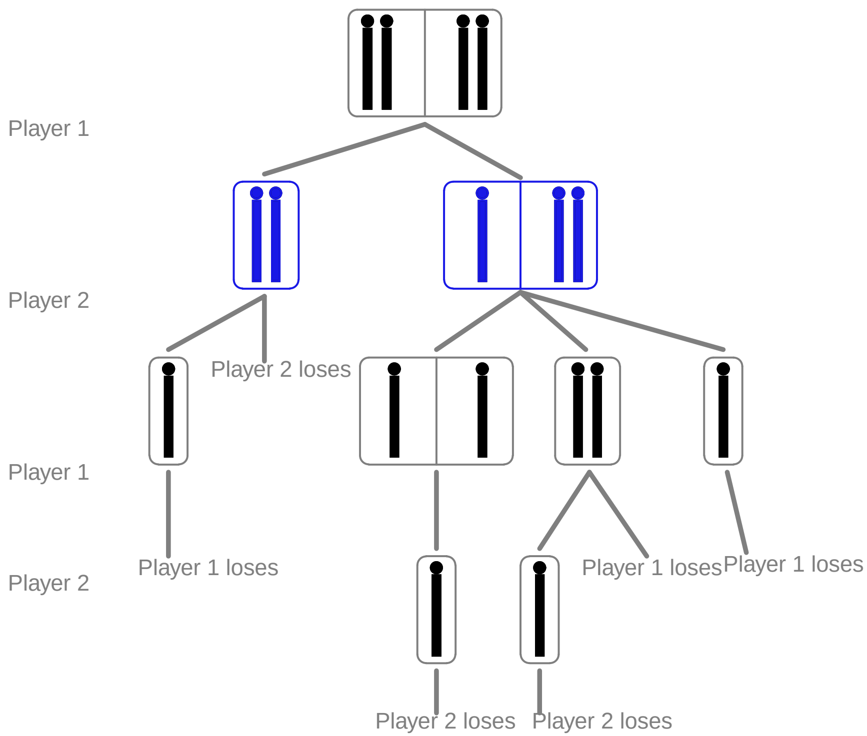

Player 2. Player 2 also has two decision points, one with two choices and one with three choices.

$(2, 2)$-Nim

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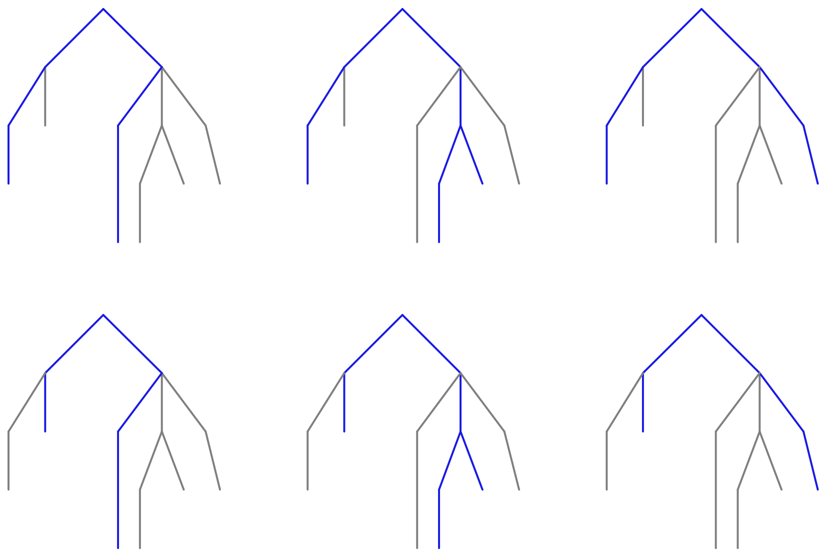
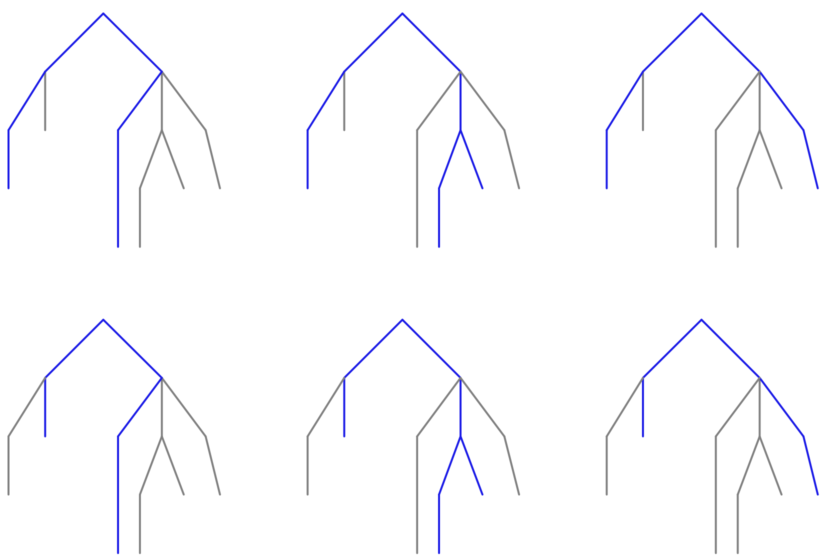


Player 2. Player 2 also has two decision points, one with two choices and one with three choices. Since Player 1 decides which decision point will be reached, all these choices can be combined with each other, giving $2 \times 3 = 6$ strategies.

$(2,2)$-Nim

Exercise 5 (a): $(2,2)$**-Nim**, all strategies for both players.



Player 2. Names for these: *eg.* use vertical bar $|$ to separate

‘what to do if Player 1 chooses the left branch’

‘what to do if Player 1 chooses the right branch’.

For the second alternative we also have to say which stack to take a match from, that with one match remaining (1), and that with two matches remaining (2). If again we simply use the number of matches to be removed we get the strategies $1|1(2)$, $1|1(1)$, $1|2(2)$, $2|1(2)$, $2|1(1)$ and $2|2(2)$.