



CS3191 Section 4

Large Games

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In fact, even the methods in the previous section will not work on such games—the game trees are so large that carrying out alpha-beta search would take far too long to return a value and thus a good move.

There are three problems which have to be solved to write such a program which we will discuss in some detail. Finally we will have a look at how [Chess-playing programs](#) developed, since Chess is the game for which the most effort has been made when it comes to writing programs.

The three problems

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Board representation and move generation. Clearly we have to think about how the board (and the pieces) are represented internally, and how the moves are to be generated. Typically, once this has been solved it can be left alone.

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Board representation and move generation.

Alpha-beta search. Despite the fact that we cannot hope to employ the minimax algorithm with alpha-beta pruning, this technique still plays a vital role in game-playing programs. There are some variants that might be implemented, and typically some effort is spent on cataloguing search results.

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Alpha-beta search.

Evaluation function. Since alpha-beta search cannot be carried out until a leaf is reached, the search stops instead at a pre-defined depth. To obtain a value for a position at this depth, a function has to be created which assigns one **based entirely on the state of the board at the time**. This is known as the 'evaluation function'.

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The faster the program, the higher the depth to which it can carry out alpha-beta search (before it has to 'guess' a value for a position), and the better it will play. Hence **speed** is of the essence when writing such programs, and is a concern for all the components mentioned above.



Representing the board and related issues

Representing the board–array

In order to illustrate our thoughts, we often use Chess as an example. However, there's no need to be familiar with the game beyond the rudiments.

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Obvious representation of a Chess board: 8×8 array.
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To generate moves: Pick piece, generate possible target fields, then:

- check target field not occupied by own piece;
- if piece is a rook, bishop, pawn or queen, check whether the way to target is empty;
- if piece is a king check that target position cannot be reached by an enemy piece in one step.

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Need:

- loop over all fields (to pick piece);
- loop over all possible target positions;
- loop to check for obstructions along the way.

Complicated, not fast.

Board representation– $0x88$

Faster: Assign a number to each square on the board given by one byte, four high bits: row; four low bits: column.

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| | | a | b | c | d | e | f | g | h | |
|---|-----------|------|------|------|------|------|------|------|------|-----|
| | | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | low |
| 8 | 0111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | |
| 7 | 0110 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | |
| 6 | 0101 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | |
| 5 | 0100 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | |
| 4 | 0011 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | |
| 3 | 0010 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| 2 | 0001 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | |
| 1 | 0000 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| | high bits | | | | | | | | | |

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|---|-----------|------|------|------|------|------|------|------|------|-----|
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| 4 | 0011 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | |
| 3 | 0010 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| 2 | 0001 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | |
| 1 | 0000 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| | high bits | | | | | | | | | |

To move one field to the left or right, just subtract or add one.

Board representation-0x88

| | | a | b | c | d | e | f | g | h | |
|---|-----------|------|------|------|------|------|------|------|------|-----|
| | | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | low |
| 8 | 0111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | |
| 7 | 0110 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | |
| 6 | 0101 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | |
| 5 | 0100 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | |
| 4 | 0011 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | |
| 3 | 0010 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| 2 | 0001 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | |
| 1 | 0000 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| | high bits | | | | | | | | | |

To move up a row, add 16, to move down a row, subtract 16.

Board representation-0x88

| | | a | b | c | d | e | f | g | h | |
|---|-----------|------|------|------|------|------|------|------|------|-----|
| | | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | low |
| 8 | 0111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | |
| 7 | 0110 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | |
| 6 | 0101 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | |
| 5 | 0100 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | |
| 4 | 0011 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | |
| 3 | 0010 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| 2 | 0001 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | |
| 1 | 0000 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| | high bits | | | | | | | | | |

Board: represented as an array with 128 entries, only 64 of which correspond to actual fields.

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| | | a | b | c | d | e | f | g | h | |
|---|-----------|------|------|------|------|------|------|------|------|-----|
| | | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | low |
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| 2 | 0001 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | |
| 1 | 0000 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| | high bits | | | | | | | | | |

This is much faster than the first version. To check whether a number i is a valid position on the board, check whether it satisfies $i \& 0x88 == 0$ (&: bitwise).

Board representation–bitboards

Idea: for each colour and piece, use a ‘bitboard’.

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Need: one 64-bit word for each piece. Operations: bit-wise—this is really fast!

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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Example: bitboard for all black pieces: bit-wise 'or' of all bitboards for black pieces.

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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Move of a piece by a row: shift the bitboard by 8.

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The white pawns:



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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Empty fields: bitboard for all pieces negated.

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The white pawns:



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

All legal moves of pawns by one field can be stored in a bitboard (similarly for all legal moves of pawns by two fields). Constant bitboards can be prepared at compile time to be available in a library.

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Only **disadvantage**: the code becomes more complicated; turning a bitboard of possible moves into a list of possible moves, for example.

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Only **disadvantage**: the code becomes more complicated; turning a bitboard of possible moves into a list of possible moves, for example.

Advantages: fast; bitboards required more than once only have to be computed once; several moves can be generated at the same time.

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Chess programs typically use a large hash table to keep track of positions that have occurred in play.

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A hash function frequently used consists of assigning to each pair, consisting of a piece and a field on the board, a large random number. The idea is that this number encodes the fact that the corresponding piece occupies the corresponding field. Then one sums up the appropriate numbers for the given position to obtain the hash key. A checksum process can be applied to make sure later that ‘the right’ position is looked up.

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This is best done by keeping a **stack** of moves with sufficient information to undo them. This is typically much cheaper than keeping a list of positions through which one has gone.



Evaluation function

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There is no such thing as 'the right' evaluation function. A big part of writing a game-playing program is to watch it play and **fine-tune** the evaluation function to improve it.

There are no hard and fast rules for what makes a good evaluation function; they are mostly based on **heuristics**.

Speed

When writing a game-playing program, speed is always an issue. Hence it pays to calculate the desired evaluation function in such a way to make the process as fast as possible.

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Let p be the current position, and e the evaluation function. Then if

$$e(p) = e_{s_1}(s_1\text{'s place in } p) + \dots + e_{s_n}(s_n\text{'s place in } p),$$

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where s_1, \dots, s_n are the pieces involved, the value of a new position resulting from one piece s being moved is

$$\text{score}(\text{move}) = e_s(s\text{'s new field}) - e_s(s\text{'s old field}).$$

Problems: For many games this kind of evaluation function is not good enough—it does not take the relative position of pieces into account.

Techniques

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It is important that an evaluation function judge any position from **both** players' point of view. Having many pieces on the board does not give White any advantage if Black is about to checkmate him!

Relevant constituent parts

Material. The number and kind of pieces on the board. Chess: Each piece has a value; Go: count number of pieces on board, Othello: same.

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Not equally useful for all games: Othello: not number of pieces is important, but their locations (corners). Player with **fewer** pieces might have better position. There are other games where the number of pieces may be irrelevant.

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Chess: count number of fields threatened by one player; Othello: count number of pieces which cannot be taken by the opponent. Calculate size, possible with weights for very important fields.

Relevant constituent parts

Material. The number and kind of pieces on the board.

Space. Influence.

Mobility. Ability to move. Having many different available moves: advantageous, *e.g.* in Othello. Chess: not clear this is useful.

Relevant constituent parts

Material. The number and kind of pieces on the board.

Space. Influence.

Mobility. Ability to move.

Tempo. Initiative. Go: one player has the **initiative**, that is, he acts, other player reacts to his moves.

Relevant constituent parts

Material. The number and kind of pieces on the board.

Space. Influence.

Mobility. Ability to move.

Tempo. Initiative. Go: one player has the **initiative**, that is, he acts, other player reacts to his moves.

Other games: try 'parity argument': often find positions where player who moves next wins/loses, can be simple to evaluate (see Nim, Connect-4).

Relevant constituent parts

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Space. Influence.

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Threats. Can one of the players capture (or threaten to capture) a piece? Connect-4, Go-Moku: can a player win in the next move?
Othello: is a player threatening to take a corner?

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Shape. How pieces on the board relate to each other. Chess: line of pawns much stronger than other grouping. Go: shape is 'territory to be'—stones outline territory which the player can defend when threatened.

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Judging shape: often very difficult. Change of shape value: incremental over time, **long-term target**. Evaluation function partially based on shape: can’t just simply add piece-based functions.

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Chess: bishop capturing a pawn on border is often trapped; Othello: sacrifice one corners in exchange for another. Deciding when a pattern applies is hard!

Fine-tuning

Deducing constraints. Chess: every piece gets a material value. Know: rook more important than pawn, *e.g.*, so value should be according. Can deduce values from experience.

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Know, *e.g.*: one rook less than two pawns and bishop, or two pawns and knight, but not less than one pawn and bishop/knight.

So: weight of a rook should be below weight of pawns and bishop, but above one pawn and bishop. Get fewer possibilities to try.

Fine-tuning

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Hand tweaking. Happens typically in practice. Programmers watch implementation play, judge which parameters to change and how. Perform the change and watch again. Reasonably fast but requires game-specific knowledge.

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Can be modified by randomly sticking with some changes which do not improve performance. 'Randomness' controlled by some probabilities (start out fairly high, become smaller as a good value is approached). Adjusted method is slower than original, but can get good values.

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Examples for learning: genetic algorithms, neural networks. Both: rather slow; main advantage: do not require game-specific knowledge. Reason for slowness: number of test games required is typically very high (commercial game programmers tried about 3000 matches to allow the program to learn—the result was worse than hand tweaking). Further problem: If opponent is too good program loses all the time and never starts learning.

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Problem with playing program against versions of itself: same lines are explored over and over. To avoid this: start the program(s) from positions a few moves into a game.



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There are some ways of fiddling with this to adjust it to the game in question. The thought is always to make it **faster** so that it can **search deeper**.

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Obvious advantage: When time runs out we give the best move found so far, and that will at least be sensible. This is known as **iterative deepening**.

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But if we use a hash table to keep track of results so far we can **estimate** a value.

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- successively increasing lower bounds for a **max node** (α);
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This technique is known as **aspiration search**.

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Good idea to include the current values of α and β in the hash table of previously searched positions.

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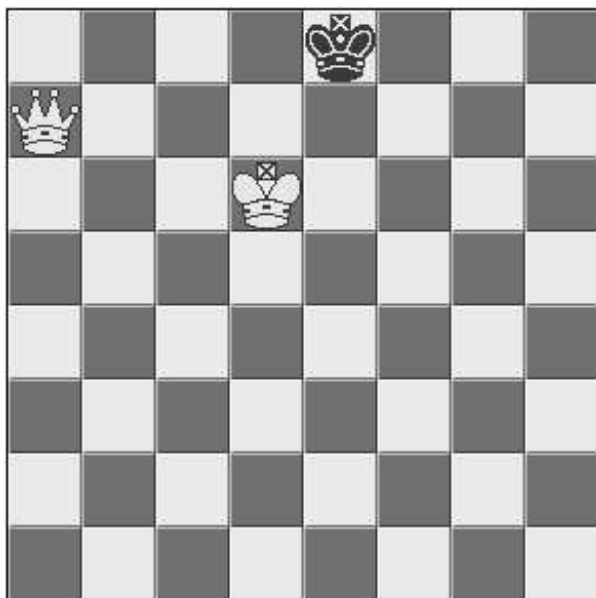
Can search the first move(s) with big window for potential value (see aspiration search), and later moves with smaller ones. This is known as **principal variation search**.

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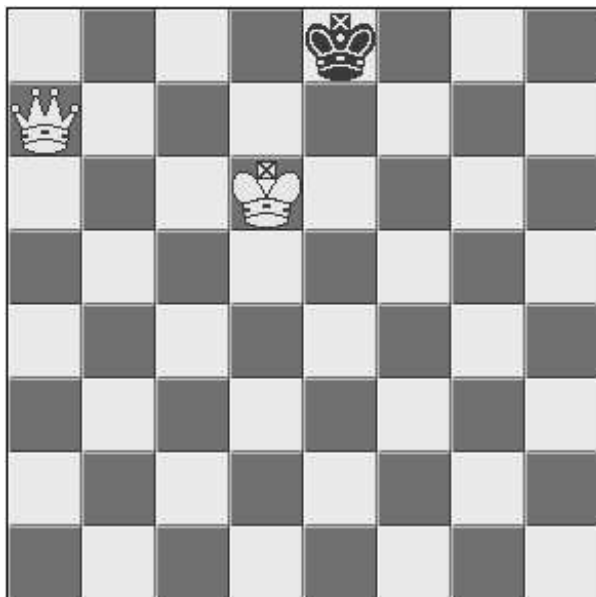
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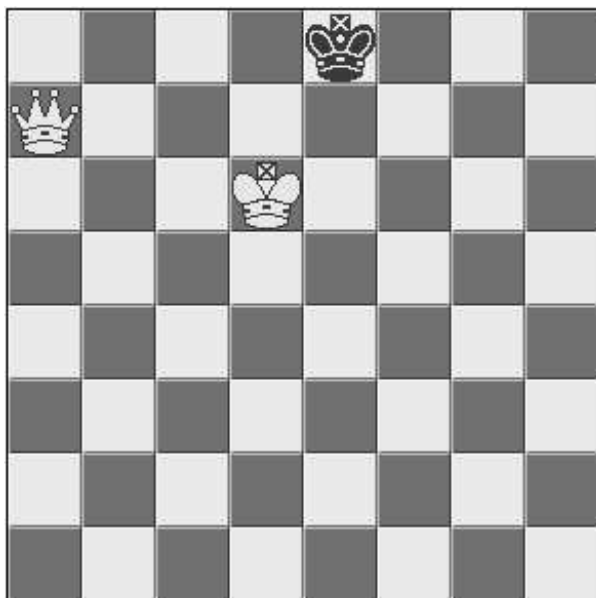
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If White moves the king to $e6$ (one field to the right) then he is still in a winning position, with Black's only valid moves being to $d8$ and $f8$.

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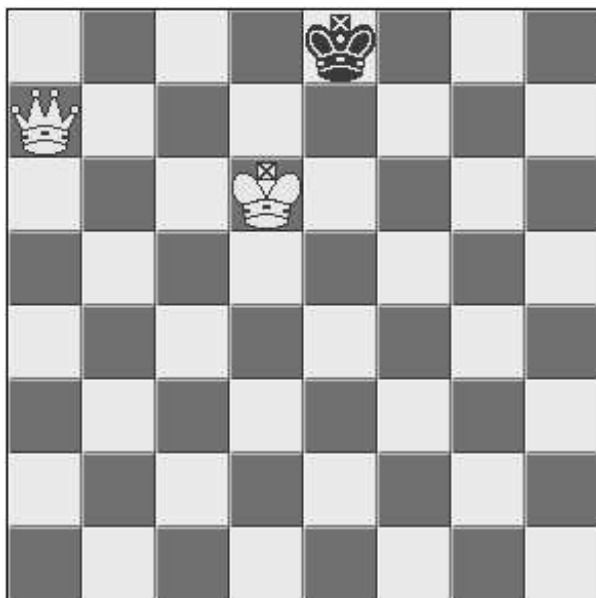
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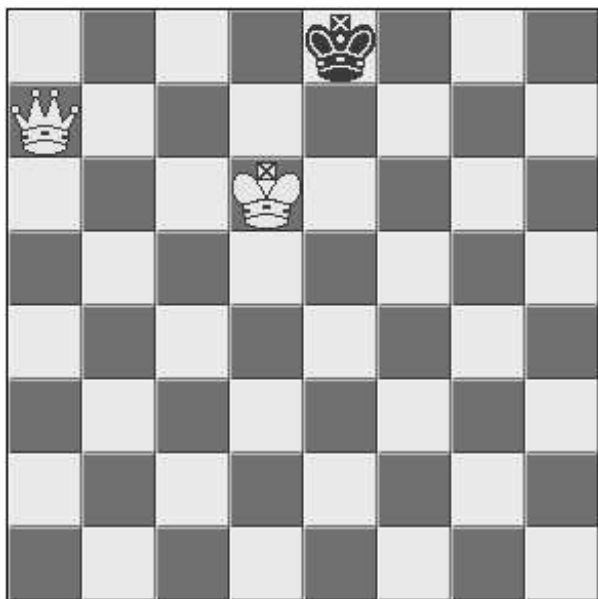


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But if Black now moves back to $e8$, we are back where we started and our program might go into a loop. This will lead to a draw since there are rules about repeating the same position.

Can avoid this by assigning slightly lower values to winning positions, for example

$1000 - \text{number of moves req'd to get win.}$

Then alpha-beta search will work properly.

The horizon effect

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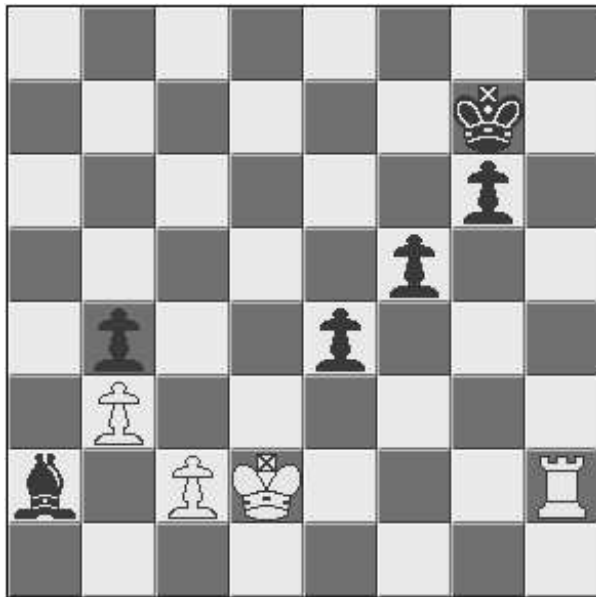
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This can lead to the program trying to fend off **bad events** (capture of its piece, for example) by keeping them below the horizon.

In order to avoid, say, the capture of one of its pieces the program may try pointless moves which merely postpone the inevitable—typically these moves do not progress the program's play.

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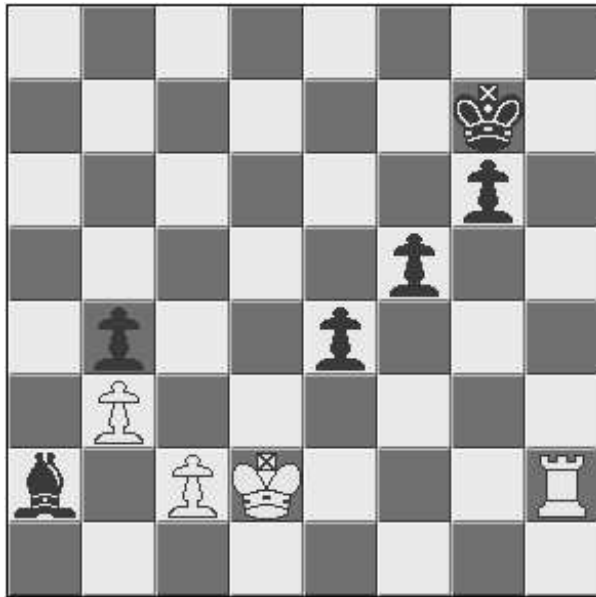
The black bishop is trapped by the white pawns. It will be captured (e.g. White rook: h2, h1, a1, a2).

If the program playing Black searches 6 moves ahead might move black pawn e4 to e3, checking the king. White has to react to this by moving the king or capturing this pawn.

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That delays capture of the bishop so that the program thinks it is safe. The program might so sacrifice all its pawns, setting itself up for a loss.

Solutions: **Add knowledge** so that program can detect when piece is trapped. Increase overall depth of search in such situations so that **horizon is widened**. Whenever piece is threatened, search to deeper level selectively.

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For example, when currently set depth is reached search deeper for all moves which are likely to lead to change of evaluation considerably (Chess: capturing moves, check moves). This is known as **quiescent search**.

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Alternatively one might increase search depth whenever the currently explored line contains a capturing move. Can only do this in a **limited way**, or the program will keep looking deeper and deeper!

Selective extension

Many games do not search to fixed depth everywhere. Instead the **select an appropriate depth**, which is greater whenever

- there is reason to believe that the current value for a position is inaccurate or
- when the current line of play is particularly important.

For example, when currently set depth is reached search deeper for all moves which are likely to lead to change of evaluation considerably (Chess: capturing moves, check moves). This is known as **quiescent search**.

Alternatively one might increase search depth whenever the currently explored line contains a capturing move. Can only do this in a **limited way**, or the program will keep looking deeper and deeper!

Many programs search deeper on what they think is the best move (see **principal variation search**).

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Mistakes or weaknesses in a program can be explored over and over (until the creator finds a chance to fix this, since these programs **don't learn**). Many tournaments between various programs seemed to be more about who could discover whose built-in faults, rather than whose program genuinely played best!



Chess-playing programs

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Shannon thought this would be a useful application for computers, and would give insights into how one makes intelligent decisions.

The first Chess programs

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1974: First world computer Chess championships. Repeated every three years.

Improvements by mid-eighties

Hash tables to keep track of positions searched to which depth, and values discovered. (Often no update of value!)

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Search to **variable depth**, depending on whether the current position is judged to be ‘tricky’ or relatively straight-forward.

Artificial Intelligence?

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Nor did any true learning take place.

The early, strong claims regarding the possibilities of AI turned to out to be vastly exaggerated. Today, Artificial Intelligence often is about search techniques and the machine learning is very different from human learning!

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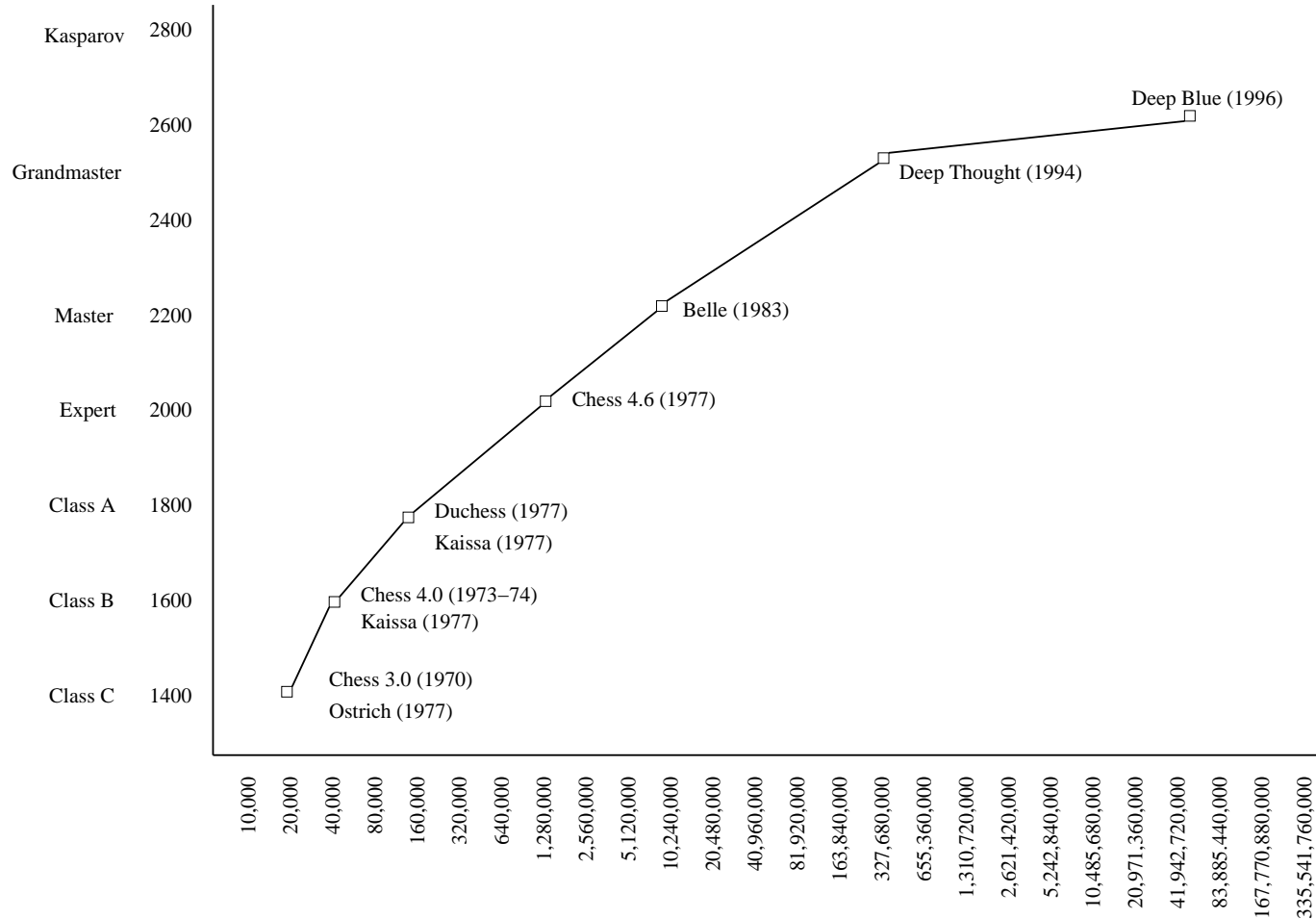
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Since late eighties: Main development has gone into specialized hardware.

Speed increases strength



Number of positions examined in three minutes, official ranking. (Note logarithmic scale along horizontal axis!) Where is **perfect play**?

Depth of search

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| | 3 | 4 | 5 | 6 | 7 | 8 | rating |
|---|----|------|------|------|------|-----|--------|
| 3 | | 4 | | | | | 1091 |
| 4 | 16 | | 5.5 | | | | 1332 |
| 5 | | 14.5 | | 4.5 | | | 1500 |
| 6 | | | 15.5 | | 2.5 | | 1714 |
| 7 | | | | 17.5 | | 3.5 | 2052 |
| 8 | | | | | 16.5 | | 2320 |

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| | 4 | 5 | 6 | 7 | 8 | 9 | rating |
|---|------|------|------|----|------|-----|--------|
| 4 | | 5 | .5 | 0 | 0 | 0 | 1235 |
| 5 | 15 | | 3.5 | 3 | .5 | 0 | 1570 |
| 6 | 19.5 | 16.5 | | 4 | 1.5 | 1.5 | 1826 |
| 7 | 20 | 17 | 16 | | 5 | 4 | 2031 |
| 8 | 20 | 19.5 | 18.5 | 15 | | 5.5 | 2208 |
| 9 | 20 | 20 | 18.5 | 16 | 14.5 | | 2328 |

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Three or four levels more of search means outclassing one's opponent!

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This table shows that the benefit is diminished as overall depth increases.

Hardware for Chess

The following table gives an overview over Chess-playing programs and the hardware they were running on.

Hardware for Chess

| Name | Year | Description |
|------------------|------|--|
| Ostrich | 1981 | 5-processor Data General system |
| Ostrich | 1982 | 8-processor Data General system |
| Cray Blitz | 1983 | 2-processor Cray XMP |
| Cray Blitz | 1984 | 4-processor Cray XMP |
| Sun Phoenix | 1986 | Network of 20 VAXs and Suns |
| Chess Challenger | 1986 | 20 8086 microprocessors |
| Waycool | 1986 | 64-processor N/Cube system |
| Waycool | 1988 | 256-processor N/Cube system |
| Deep Thought | 1989 | 3 2-processor VLSI chess circuits |
| Star Tech | 1993 | 512-processor Connection Machine |
| Star Socrates | 1995 | 1,824-processor Intel Paragon |
| Zugzwang | 1995 | 96-processor GC-Powerplus distributed system (based on the PowerPC) |
| Deep Blue | 1996 | 32-processor IBM RS/6000 SP with 6 VLSI chess circuits per processor |

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1997: Rematch, ending 2.5 to 3.5, Kasparov makes mistake in final and deciding match.

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Chess-playing programs have done very little to improve our understanding of how humans think and make decisions.

Other games

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Go-playing programs currently are way below even good amateurs, let alone professionals.

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- **Alpha-beta search** assigns a value to a position by searching the game tree below it and eventually applying the evaluation function. Searching to greater depth will result in a better program, so any gain in speed goes into searching to a greater depth. There are many tricks to try to only search the relevant parts of the game tree; in particular ordering moves to search the most promising ones first.

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- **Alpha-beta search** assigns a value to a position by searching the game tree below it and eventually applying the evaluation function.
- Most effort so far has gone into creating Chess-playing programs. They have profited from faster hardware, and many improvements have been made which are very Chess-specific: better heuristics, opening and endgame libraries, and the like.