Forty-five minutes

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science

12/11/18

Time: 11.00

Please answer all TWO Questions

This is a CLOSED book examination

The use of electronic calculators is <u>not</u> permitted.

1. a) Consider the following function:

$$f: [1, \infty) \longrightarrow [0, 1]$$
$$x \longmapsto x^{-1}.$$

   Is this function injective? Is it surjective? Justify your answers. (5 marks)

   b) Consider the binary operation on non-empty binary strings defined as follows: To obtain $s \circledast s'$, take s and replace its last symbol with the last symbol of s'. For example we have
   $$01001 \circledast 110 = 01000.$$

   Is this operation associative? Is it commutative? (5 marks)

2. a) Consider the formula
   $$\neg(Q \to P) \wedge \neg(\neg R \to P).$$

      i) Construct the truth table for this formula. (3 marks)
      ii) Read off the disjunctive normal form of the formula from the truth table.
      (1 mark)

   b) For **one** of the following notions explain one main use in propositional logic.
   (2 marks)

      i) binding precedence
      ii) Substitution Theorem

   c) Consider this propositional formula.

   $$(\neg A \to B) \vee \neg\big(\neg B \to (A \wedge C)\big)$$

      i. Use our CNF algorithm to transform the formula into conjunctive normal form.
      ii. Simplify your answer as much as possible.

   Justify all the steps in your derivations. (4 marks)