# Exam Performance Feedback Form

## COMP11120, Semester 2

2017/2018

First some general remarks. 193 students sat the exam. The average was 61% (or 36.5 out of 60 marks), with a median of 63 %. Sixty-nine students achieved a first class mark, with a maximum of 56 marks out of 60. Nineteen students had a failing mark, with only six of these a mark below 18, which is less than 30% of the available total, and in turn none of those had a mark below 10.

If we look between the extremes than twenty-two students had a mark from 40% to 49%, forty-three from 50% to 59%, and thirty-eight one from 60% to 69%.

The overall average is an increase on previous years. We felt that most students knew the material well and could produce good answers. The question with the lowest average was suffering from the fact that almost all students answered part a) incorrectly for the first relation, and part c) also had an unexpectedly low average. The average for this question could quite easily have been 7 or 10% higher. 191 students sat the exam.

**Question 1.** Overall the answers to this question were good, with part b) being particularly well answered overall.

a) This was well answered by most. A fair number of students lost one of the four marks because there were no justifications given (typically only the use of the induction hypothesis was highlighted). A handful of students had problems deriving the statement that needs proving in the step case. Also a handful of students wrote the statement they needed to prove and went on to arrive at a correct statement—if they did not note that they need the reverse implication they lost a mark. A lot of students only seemed to calculate one side of the base case. Quite a few students defined two functions, typically named $L$ and $R$, for the left hand side and the right hand side of the expression (depending on $n$), and their proofs often were very well organized, so maybe that would work for others.

b) In this question the Java function should really have been called **dltodd** as well, and I did receive a query about this in the exam. This does not seem to have caused any problems. This question was very well answered with many students receiving all ten marks. Marks were sometimes lost because of mistakes in giving the various cases (of which more below), not giving any justifications in the proof, or forgetting to write down the base case. Many students only evaluated the base case to 0 without specifically saying that this is an even number, but they still received the mark allocated for it. Students who made a syntax error in their definition might lose another mark because they then had to use something in their proof which was not the step case of the sum operator. I'd suggest that students sitting future version of this exam ensure that they really do use the given function as it is defined on the sheet—if they can't they've made a mistake in their own definition.

   The base case needs to return the empty list [], the Java syntax of which is **null** (or returning the input list l also works). There are two step cases: For a list $s : l$ we need to check whether $s$ is even or odd. If the former then we want $s :$ dltodd $l$, if the latter, dltodd $l$.

   In the Java code as given I left a large space for the **else** case, and almost all students realized that there were two cases required here. They are

```
{
```

```
        if (l.value % 2 == 0)
            return new List (l.value, dltodd (l.next));
        else
            return dltodd (l.next);
        }
    }
```

Some students could not remember the correct syntax for lists, and a mistake often made was to either not use **new** List where required (in the first return case), or to use it where it's not wanted (in the second return case). Some students had problems with how to determine whether l.value is even.

Some students defined auxiliary lists and put those in the return clause—if they did that correctly they received full marks.

Students didn't have much prior experience with writing down a proof with a case distinction, but almost all managed to communicate what was wanted.

c) There was a typo in this question—there is a $\neg$ missing in front of $A_n$. Again most students either didn't notice the typo, or they automatically corrected it. The question was a bit of a wild card, since we haven't done anything quite like this in the examples classes. The great majority of students worked out that the proof is an induction over $n$. The statement does hold for $n = 1$ (and even $n = 0$) and some students started their induction at 1, which is fine. Otherwise the base case is the given De Morgan law, and for the step case one needs to use associativity to bracket off the first $n$ terms, then use the De Morgan law, and then the induction hypotheses and associativity again to obtain the desired right hand side. If students gave no justification they lost a mark, but I was generous if they didn't specifically mention associativity (or erroneously called it commutativity).

d) I was looking for the definition (or something equivalent), giving examples or writing something informal did not gain any marks. Any statement that is in the notes and is equivalent to the definition got both marks. Some students only gave the definition of one of the properties (countable or infinite), and they received one of the marks. The shortest correct answer is that it's a set that has a bijection to the natural numbers.

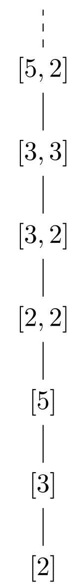
**Question 2.** Overall this was quite well answered, but disappointingly many students received no marks for 2.a)(i). Part c) had a low-ish average mark because students did not know at least some of the terms (or they could not give their properties correctly). In part d) many students could see that the statement is true but they could not write down a formal argument.

a) The first relation given is not an equivalence relation since it is not symmetric. There are lots of counter examples. The first mistake students made was not to work out that if we know that for every column vector of $\underline{B}$ there exists a column vector of $\underline{A}$, this really does not tell us anything about whether there's a column vector of $\underline{B}$ for every one of $\underline{A}$. The simplest counterexample is a $(1 \times 1)$ matrix: We have that the matrix $(1)$ is related to $(0)$, but not the other way round. One mark was given for correctly determining that this is not an equivalence relation, one mark for noting that it is not symmetric, and one for giving a correct reason that it is not symmetric. Nothing else written gained any marks.

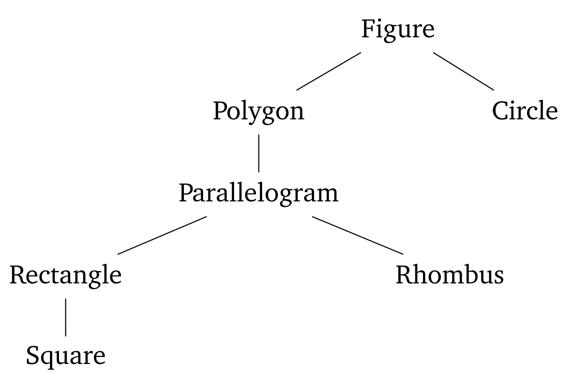
The second relation was correctly determined to be an equivalence relation by everybody who answered Question 2. The only way some students lost marks is that they did not give enough detail for the argument. Saying 'it clearly is transitive' is not sufficient.

b) There was a typo in this question, where ‘time’ occurred where ‘team’ should have been, but everybody interpreted the question correctly. The first relation given is not a partial order since it is neither reflexive nor transitive and almost all students were able to determine this. For all three marks it was sufficient to give a correct argument that one of these properties fails.

The second relation is a partial order (in fact, it is a total order as noted by some students). It is relatively easy to argue that it is reflexive and transitive, the only tricky part is anti-symmetry. Some students did not understand that it is not sufficient to note that if $l$ is related to $l'$ and $l'$ is related to $l$ then the product of all numbers in $l$ must be equal to the product of all numbers in $l'$—one needs to show that the two lists are equal. A lot of students said that because only prime numbers appear in the lists we must have equality, but the lists $[2, 3]$ and $[3, 2]$ have the same product, so to get the mark assigned for anti-symmetry more needed to be stated here, and a lot of students did not give a correct argument. There were also quite a few missing or incorrect Hasse diagrams. I was a bit surprised that I hardly saw any Hasse diagrams that contained repeated elements (such as $[2, 2]$). Some students drew Hasse diagrams that didn’t have any lists, and they did not get the mark assigned. I give my Hasse diagram on the right. Some students had the prime numbers in the list ordered the other way round, and that did not matter for getting the marks.




c) The relation we want here was clearly defined as the ‘is an instance of’, which agrees with what one would want to be a Java ‘is a subclass of’ relation. For some reason a lot of students drew the Hasse diagram upside down. They still got marks for parts ii) and iii) if they correctly determined the required elements. It seems easiest to give the model answer.




There is another answer I gave as correct, since every Square is also a Rhombus. One can’t have a Java class that is a subclass for more than one other class, but I didn’t think it was clear enough in the question as asked, so the corresponding solution was also marked as correct. There is one maximal element which is also the greatest element, in the given Hasse diagram there are three minimal but no least element, and the answer to iv) is Parallelogram.

The issue some students had with the terms given has made me enquire whether students could be allowed to bring an English-English dictionary—the current University policy doesn’t really specify either way. This is now being pursued within the University for future exams.

d) Most students who attempted to answer this part seemed to be able to tell why the given property holds, but a lot of them could not write a formal argument. The function $f$ is defined for natural numbers, so writing $f[0]$ does not make sense (I saw that a lot). A lot of students tried to argue that the elements of the equivalence classes [0] and [4], say, are mapped to 0 (they often wrote [0]), and so on, but their arguments had either holes or did not respect the given types. The answer I was looking for is the one that notes that if two numbers $m$ and $n$ are equal $\bmod 6$ then we can find unique numbers $k$, $l$, and $i$, where $i$ is in $\{0, 1, 2, 3, 4, 5\}$, such that $m = 6k + i$ and $n = 6l + i$, and one can then compute that the result of applying $f$ to either number is $i \bmod 3$. For full marks each step needed a justification. Quite a few students wanted to use that for all $m \in \mathbb{N}$,

$$m(\text{mod}3) = (m(\text{mod}6))(\text{mod}3),$$

but that requires proving.

**Question 3.**

General remark: Overall students did very well on this question and I was very pleased with the quality of the answers.

The following small adjustment was made to the mark distribution to reflect the effort and detail required for the answers (and because it was intended this way anyway):

- 3 marks for Question 3b)1) (instead of 4)
- 5 marks for Question 3c) (3 plus 2, instead of 2 plus 2 = 4)

a) Part i) posed no problem but it was noticeable how many students struggled to give a geometric interpretation for the task in part ii). Only a handful of students got the answer I was looking for: geometrically, the task of finding $\underline{v}$ is the task of finding the definition of the third side of a triangle in which two sides are given by the vectors $\underline{a}$ and $\underline{b}$. I was generous in the marking this part, but the answer needed to have a geometric aspect. For example, $\underline{v}$ is the additive inverse of $\underline{a} + \underline{b}$ did not earn a mark. An appropriate drawing also earned a mark.

b) I was generally very happy with the answers. The question required being able to multiply matrices, which almost everybody could do.

For part ii) it was sufficient to give a counter-example using instances of the matrices from part i). This would have been easiest, but other, correct counter-examples were accepted.

c) This question was answered either reasonably well, or not at all (by a small number of students).

In part i) most students proceeded by formally showing that $T$ is a linear transformation. Some students then forgot to compute the matrix representation of $T$. The formal proof can be avoided if the matrix representation is computed directly. If the matrix is computed via the application of $T$ to the unit vectors, then it needs to be checked that the matrix does indeed define $T$.

In part ii) there were several ways of proceeding. Most students managed to give a correct counter-example for one of the properties of linear transformations. Easiest was to show the zero vector is mapped to a non-zero vector. Easy marks were sometimes lost for not explicitly stating the answer to the question, i.e., that $S$ is not a linear transformation.

Marks were given for correct answers even if they were guesses.

d) As expected this was the hardest part. Common mistakes were:

- Arguing $\underline{B}^2 = \underline{0}$ implies $\underline{B} = \underline{0}$. This is *not* true; simple counter-examples can be given.
- Arguing $\underline{AB} = \underline{A}$ implies $\underline{B} = \underline{I}$. This is *not* true either.

Many students attempted to give a component-wise argument, but this is very hard (almost impossible I would say).

A short component-free argument can be given which exploits the laws of matrix multiplication and the given two equations, which a good number of students did. There is no need to use addition.

e) Almost everyone got part i) right. One or two students wrote the vector equation representation of the system, but this is not what I was looking for.

Part ii) was generally answered very well. Marks were mostly lost due to to small mistakes in the calculations. The question explicitly required the use of our Gaussian elimination method. If other methods were used, the marking scheme was: 2 marks for correct answers, otherwise 0.

Part iii) required two solution *sets* to be given. *Wrong* answers to the first part included:

- no solution set for $\underline{Ax} = \underline{b}$.
- since the empty set has no elements it does *not* make sense to write $x \in \emptyset$.
- $S = \{(\emptyset)\}$ or $S = \{\emptyset\}$ is wrong.

The correct answer was: the solution set is the empty set, i.e., $S = \emptyset$.

For computing the solution set of $\underline{Ax} = \underline{0}$, students who recognised that their answer to part ii) could be re-used saved a lot of time and calculations. The augmented matrix of $\underline{Ax} = \underline{0}$ is identical to the augmented matrix of $\underline{Ax} = \underline{b}$ except that the right-most column is replaced by all zeros. Everybody solving the equations directly without using Gaussian elimination would not have recognised this. It is therefore worth learning the Gaussian elimination method!