Unification with Abstraction and Theory Instantiation in Saturation-based Reasoning

Giles Reger[1], Martin Suda[2], and Andrei Voronkov[1,2,3]

[1]School of Computer Science, University of Manchester, UK
[2]TU Wien, Vienna, Austria
[3]Easychair

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This is a (slightly) extended version of the talk given at TACAS 2018

Thank you to Martin Suda for preparing the slides
I also stole some from Martin Riener

All mistakes are my own

Introduction

What is Vampire:

- Automatic Theorem Prover (ATP) for first-order logic
- Main paradigm: superposition calculus + saturation
- Also:
  - efficient term indexing
  - use of incomplete strategies
  - strategy scheduling
  - and theory reasoning

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Reasoning with Theories

- huge application demand:
  - program analysis, software verification, . . .
- inherently hard, especially with quantifiers !

Now available! `http://vprover.github.io` (License applies)

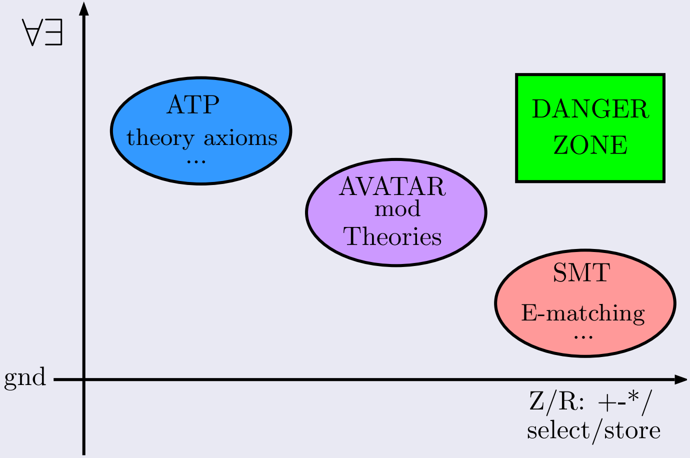
Competitions

- Regular successful participation at the CASC competition
- Since 2016 also participating in SMT-COMP

But it would be nice to get more 'real' benchmarks to demonstrate that these results generalise – SMT-COMP is better than CASC for this. Submit your problems to the libraries (if allowed)!

Reasoning with Quantifiers and Theories

Two Dimensions of Complexity



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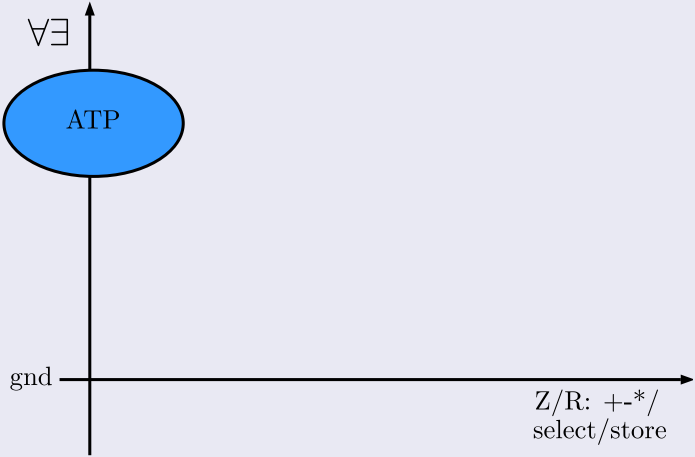

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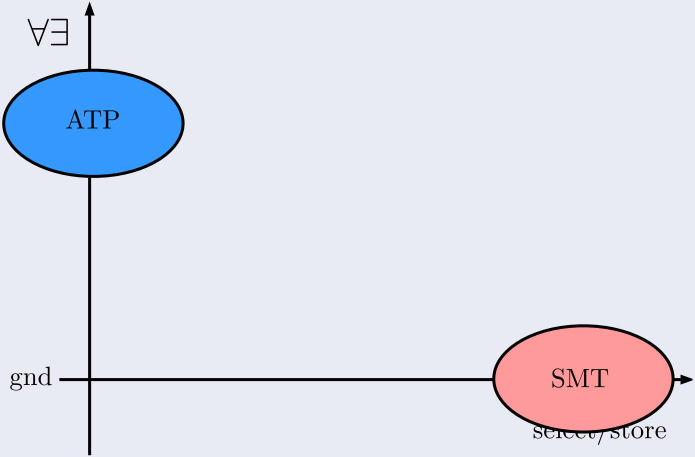

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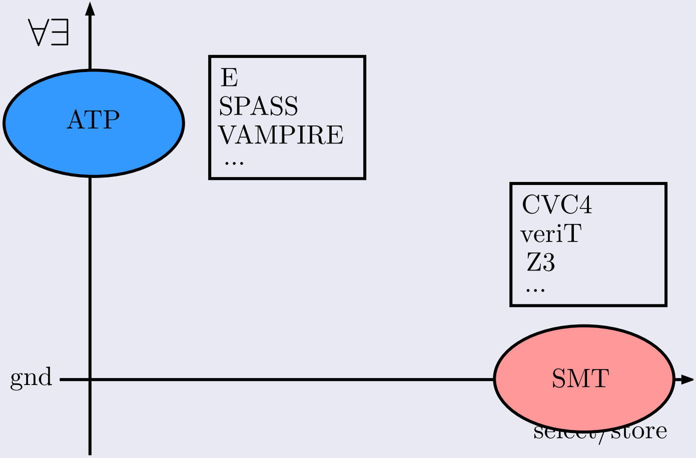

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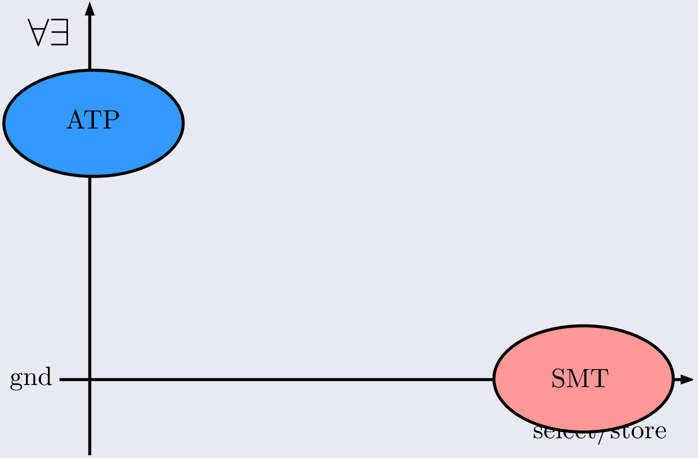

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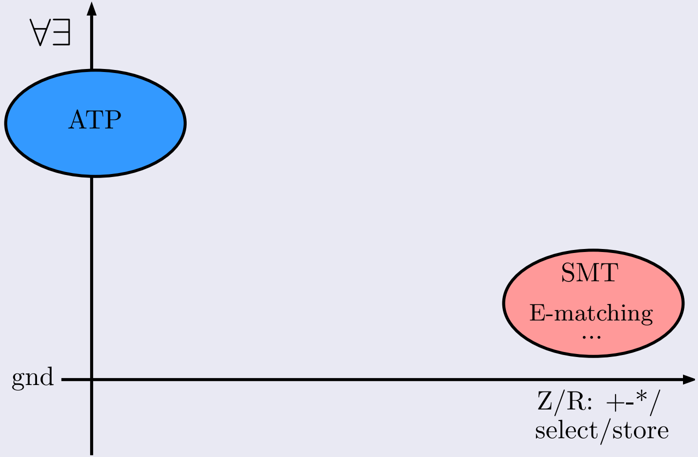

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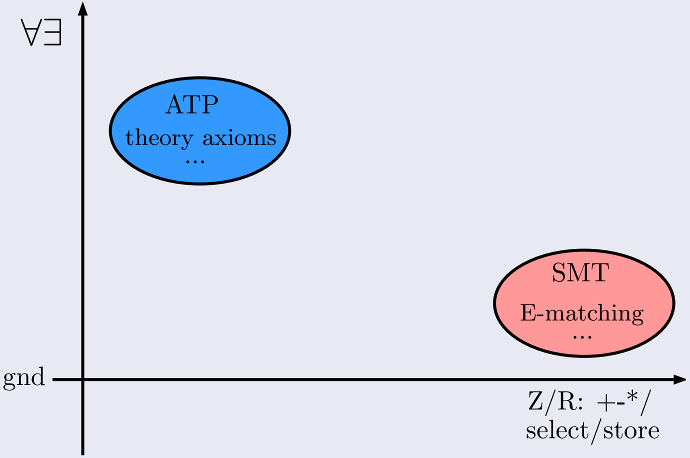

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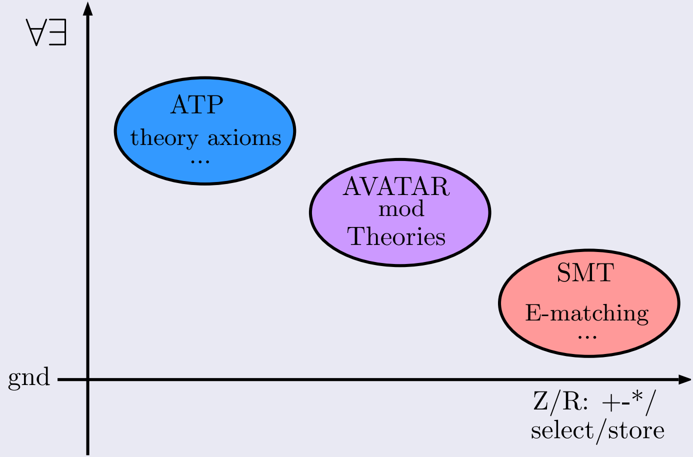

Our Paper in One Slide

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- a lazy approach to abstraction

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- a lazy approach to abstraction
- new constrains can be often “discharged” by 1.

Outline

Theorem Proving Pipeline in One Slide

Standard form of the input:

$$F \quad := \quad (Axiom_1 \wedge \ldots \wedge Axiom_n) \rightarrow Conjecture$$

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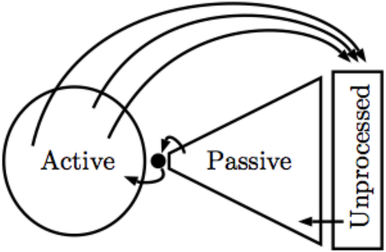
$$\mathcal{S} \quad := \quad \{C_1, \ldots, C_n\}$$

3. saturate $\mathcal{S}$ with respect to the superposition calculus

aiming to derive the obvious contradiction $\perp$

Saturation = fixed-point computation

Given Clause Algorithm:



- set of active clauses is stored in indexing structures
- passive works like a priority queue
- the process is "explosive" in nature

Superposition rule

$$\frac{\underline{l \simeq r} \vee C_1 \quad \underline{L[s]_p} \vee C_2}{(L[r]_p \vee C_1 \vee C_2)\theta} \quad \textit{or} \quad \frac{\underline{l \simeq r} \vee C_1 \quad \underline{t[s]_p \otimes t'} \vee C_2}{(t[r]_p \otimes t' \vee C_1 \vee C_2)\theta},$$

where $\theta = \mathrm{mgu}(l, s)$ *and* $r\theta \not\succeq l\theta$ *and, for the left rule* $L[s]$ *is not an equality literal, and for the right rule* $\otimes$ *stands either for* $\simeq$ *or* $\not\simeq$ *and* $t'\theta \not\succeq t[s]\theta$

Controlling the Growth of the Search Space

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Saturation up to Redundancy

- redundant clauses can be safely removed
- subsumption - an example reduction:

remove C in the presence of D such that $D\sigma \subset C$

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Completeness considerations

Outline

Basic Support for Theories

- Normalization of interpreted operations, e.g.

$$t_1 \geq t_2 \rightsquigarrow \neg(t_1 < t_2) \qquad a - b \rightsquigarrow a + (-b)$$

- Evaluation of ground interpreted terms, e.g.

$$f(1+2) \rightsquigarrow f(3) \qquad f(x+0) \rightsquigarrow f(x) \qquad 1+2 < 4 \rightsquigarrow \mathit{true}$$

- Balancing interpreted literals, e.g.

$$4 = 2 \times (x+1) \rightsquigarrow (4 \text{ div } 2) - 1 = x \rightsquigarrow x = 1$$

- Interpreted operations treated specially by ordering
  (make interpreted things small, do uninterpreted things first)

Adding Theory Axioms

$$
\begin{array}{cc}
x + (y + z) = (x + y) + z & x + 0 = x \\
x + y = y + x & -(x + y) = (-x + -y) \\
- - x = x & x + (-x) = 0 \\
x * 0 = 0 & x * (y * z) = (x * y) * z \\
x * 1 = x & x * y = y * x \\
(x * y) + (x * z) = x * (y + z) & \neg(x < y) \vee \neg(y < z) \vee \neg(x < z) \\
x < y \vee y < x \vee x = y & \neg(x < y) \vee \neg(y < x + 1) \\
\neg(x < y) \vee x + z < y + z & \neg(x < x) \\
x < y \vee y < x + 1 \ (\mathit{for\ ints}) & x = 0 \vee (y * x)/x = y \ (\mathit{for\ reals})
\end{array}
$$

- a handcrafted set
- subsets added based on the signature
- ongoing research on how to tame them [IWIL17]

AVATAR modulo Theories (since 2015)

The AVATAR architecture [Voronkov 2014]

- modern architecture of first-order theorem provers
- combines saturation with SAT-solving
- efficient realization of the *clause splitting rule*

$$\forall x, z, w.\ \underbrace{s(x) \vee \neg r(x, z)}_{\textit{share } x \textit{ and } z} \vee \underbrace{\neg q(w)}_{\textit{is disjoint}}$$

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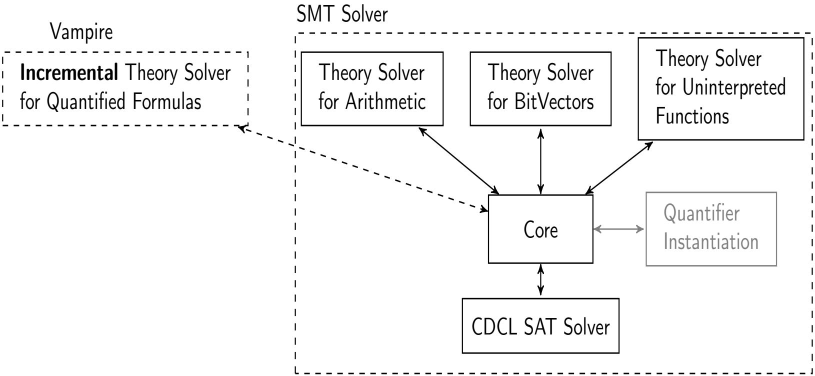
AVATAR modulo Theories [Reger et al. 2016]

- use an SMT solver instead of the SAT solver
- sub-problems considered are **ground-theory-consistent**
- implemented in Vampire using Z3

One Slightly Imprecise View of AVATAR



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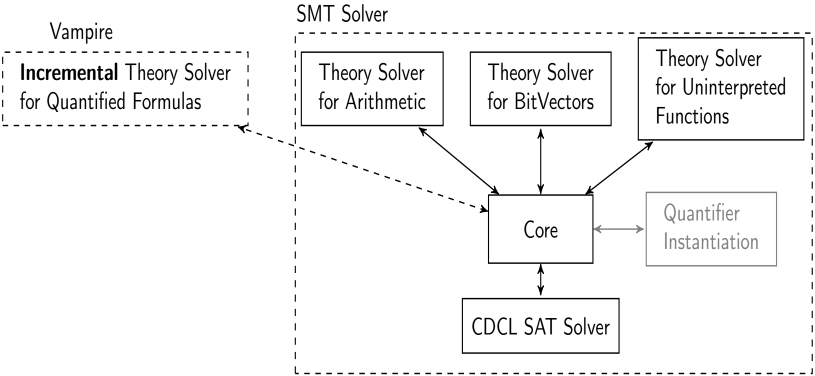

. . . but please remember: Vampire is the boss here!

Outline

Does Vampire Need Instantiation?

Example

Consider the conjecture $(\exists x)(x + x \simeq 2)$ negated and clausified to

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It takes Vampire 15 s to solve using theory axioms deriving lemmas such as

$$x + 1 \simeq y + 1 \vee y + 1 \leq x \vee x + 1 \leq y.$$

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Heuristic instantiation would help, but normally any instance of a clause is immediately subsumed by the original!

Another Example

Example

Consider a problem containing

$$14x \neq x^2 + 49 \vee p(x)$$

It takes a long time to derive $p(7)$ whereas if we had guessed $x = 7$ we immediately get

$$14 \cdot 7 \neq 7^2 + 49 \vee p(7)$$

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Implementation:

- Collect relevant pure theory literals $L_1, \ldots, L_n$
- Run an SMT solver on the ground $\neg P[\mathbf{x}] = \neg L_1 \wedge \ldots \wedge \neg L_n$
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Problems with Abstraction

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- Abstract to

$$z \neq 14y \vee r(z)$$
$$u \neq x^2 + 49 \vee \neg r(u) \vee p(x)$$

- (We discuss abstraction more later)

- Instantiation undoes abstraction:

$$\begin{array}{cl} p(1,5) & \\ \rightsquigarrow & \qquad \text{abstract} \\ x \neq 1 \vee y \neq 5 \vee p(x,y) & \\ \rightsquigarrow & \qquad \text{instantiate} \\ p(1,5) & \end{array}$$

Updated Rule

$$\frac{P \vee D}{D\theta} \text{ theory instance}$$

- $P\theta$ unsatisfiable in the theory
- P pure
- P does not contain trivial literals

A literal is trivial if

- Form: $x \neq t$ (x not in t)
- Pure (only theory symbols)
- x only occurs in other trivial literals or other non-pure literals

Flavours of Theory Instantiation

Example

A possible instance: $x \not\approx 1 + y \vee p(x, y) \implies p(1, 0)$ vs.
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Three options for `thi`:

- `strong`: Only select <u>strong</u> literals where a literal is strong if it is a negative equality or an interpreted literal
- `overlap`: Select all strong literals and additionally those theory literals whose variables overlap with a strong literal
- `all`: Select all non-trivial pure theory literals

Interacting with the SMT solver

Recall that we collect relevant pure theory literals $L_1, \ldots, L_n$ to run an SMT solver on $T[\mathsf{x}] = \neg L_1 \wedge \ldots \wedge \neg L_n$

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Evaluation may fail:

- result out of Vampire's internal range
- result is a proper algebraic number

Unification with Abstraction

Recall the abstraction rule

$$L[t] \vee C \implies x \not\approx t \vee L[x] \vee C,$$

where L is a theory literal, t a non-theory term, and x fresh.

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then resolve to get

$$u \not\simeq 14y \vee u \not\simeq x^2 + 49 \vee p(x)$$

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Now the whole Superposition calculus can be extended to use Unification with Abstraction instead of standard unification

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Four option to choose from:

- `interpreted_only`: only produce a constraint if the top-level symbol of both terms is a theory-symbol,
- `one_side_interpreted`: only produce a constraint if the top-level symbol of at least one term is a theory symbol,
- `one_side_constant`: as `one_side_interpreted` but if the other side is uninterpreted it must be a constant,
- `all`: allow all terms of theory sort to unify and produce constraints.

Implementation

Extend Substitution Trees to generate constraints when two things don't match

Also need to lookup next node by (interpreted) sort not just head symbol (a bit of book-keeping overhead)

Need to get these constraints to work with how Vampire implements backtracking and variable renaming (was the hardest part)

Outline

Comparing New Options:

- `uwa`, `thi`, `fta`

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Methodology:

- take hard, but solvable problems
- randomize (problem, other options), sample the sub-cube

Experiment with Vampire

Comparing New Options:

- `uwa`, `thi`, `fta`

Methodology:

- take hard, but solvable problems
- randomize (problem, other options), sample the sub-cube

For this experiment:

- 24 reasonable combinations of option values: `fta`, `uwa`, `thi`
- approx. 100 000 runs in total

Comparison of Three Options

fta	uwa	thi	solutions
on	off	all	252
on	off	overlap	265
on	off	strong	266
on	off	off	276
off	all	all	333
off	all	overlap	351
off	all	strong	354
off	one_side_interpreted	all	364
off	all	off	364
off	one_side_constant	all	374
off	interpreted_only	all	379
off	one_side_interpreted	overlap	385
off	one_side_interpreted	strong	387
off	off	all	392
off	one_side_constant	strong	397
off	one_side_constant	overlap	401
off	interpreted_only	overlap	407
off	one_side_interpreted	off	407
off	interpreted_only	strong	409
off	one_side_constant	off	417
off	off	overlap	428
off	interpreted_only	off	430
off	off	strong	431
off	off	off	450

Contribution to Strategy Building

SMT-LIB

Logic	New solutions	Uniquely solved
ALIA	1	0
LIA	14	0
LRA	4	0
UFDTLIA	5	0
UFLIA	28	14
UFNIA	13	4

TPTP

Category	New solutions	Uniquely solved
ARI	13	0
NUM	1	1
SWW	3	1

Ongoing and Future Work

More theories

- Currently just implemented for arithmetic
- Currently working on arrays and datatypes
- Higher-order logic as a theory?

Handling uninterpreted symbols

Tighter connection to AVATAR modulo theories

- Incorporate background knowledge about current model

More general theory instantiation

- More than one solution (inequalities)
- All solutions?
- More 'general' solutions?

Conclusion

Two new techniques for reasoning with theories and quantifiers

- theory instantiation
- unification with abstraction

Experiment with Vampire: success on previously unsolved problems

Watch this space