# Information theoretic feature selection in multi-label data through composite likelihood

#### Konstantinos Sechidis, Nikolaos Nikolaou, and Gavin Brown

School of Computer Science University of Manchester



The University of Manchester

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Male, Person, Motorbike, Vehicle Building



Female, Person, Building



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Rabbit, Animal Car, Vehicle

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• Common characteristic of these domains: Large number of features

# Feature Selection

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- Filter Assumption: model and feature selection are independent

# Feature Selection via Likelihood Maximization

 Brown et al. (JMLR 2012) unified many heuristic information-theoretic filter criteria for feature selection

Conditional Likelihood Maximization under model



# Feature Selection via Likelihood Maximization

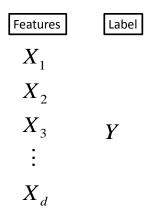
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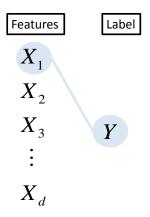
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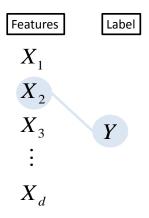


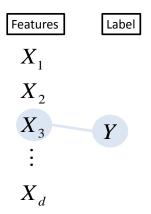
• Negative log-likelihood asymptotically decomposes into 3 terms:  $\lim_{N \to \infty} -\ell = \text{ model term} + \text{ feature selection term} + \text{ Bayes error}$ 

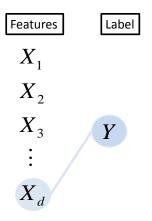
feature selection is mutual info  $I(X_{\theta}; Y)$ 

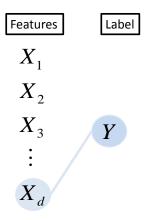






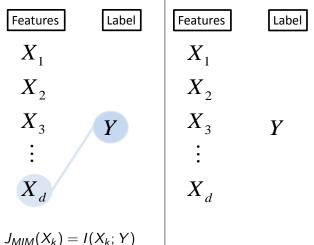




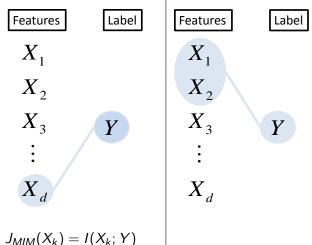


 $J_{MIM}(X_k) = I(X_k; Y)$ 

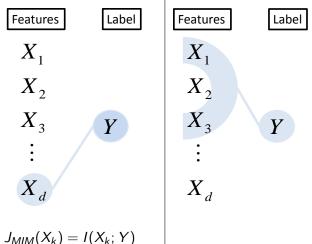
Feature space independence assumptions: Full: | Partial



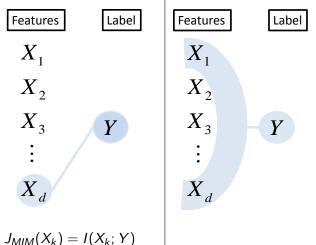
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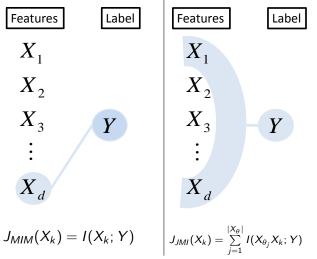
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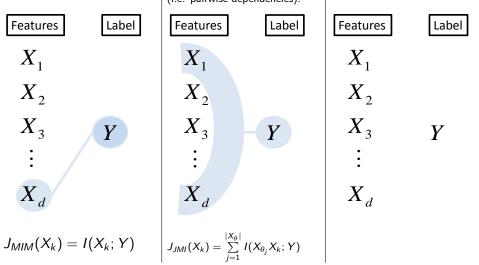
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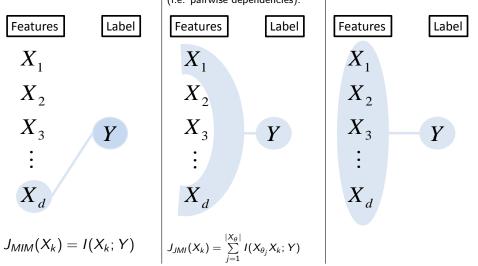
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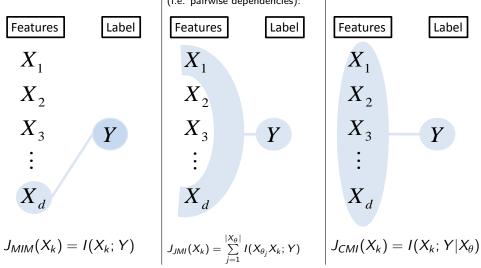
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# Extending Framework to Multi-label Setting

#### • Next, extend to multi-label where Y is q-dimensional



Man, Hat, Person

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- What independence assumptions can we make in label space?
- In this work we examined:
  - Binary Relevance (BR) vs Label Powerset (LP)

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	Animal	Building	Vehicle		у
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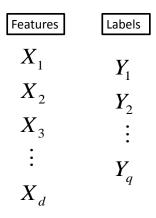
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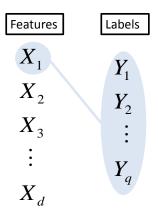
• Pros: dependencies among labels are accounted for

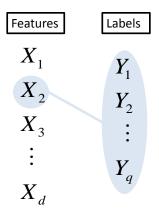
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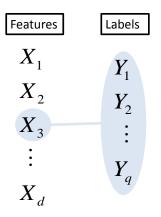
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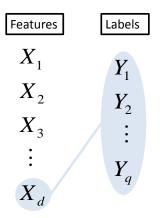
- Pros: dependencies among labels are accounted for
- Cons: probability estimates unreliable (curse of dimensionality)

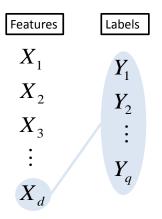






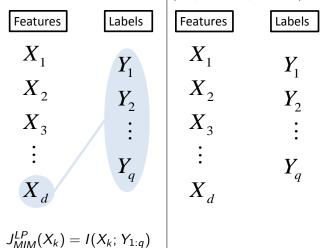






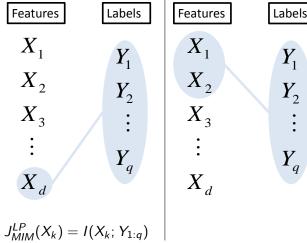
 $J_{MIM}^{LP}(X_k) = I(X_k; Y_{1:q})$ 

Feature space independence assumptions: Full: | Partial



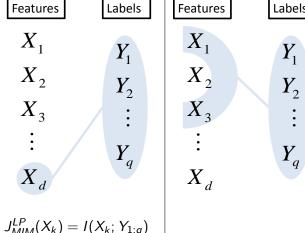
Feature space independence assumptions: Full: Partial

(i.e. pairwise dependencies): Features Labels Features Labels  $X_1$  $X_1$  $Y_1$  $Y_1$  $X_{2}$  $X_{2}$  $Y_2$  $Y_2$  $X_3$  $X_3$ :  $Y_q$ 

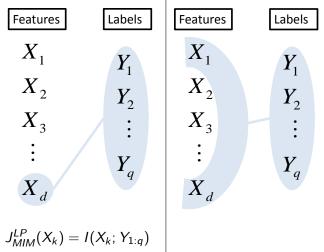


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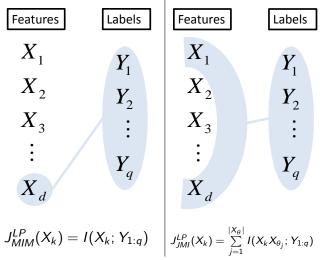
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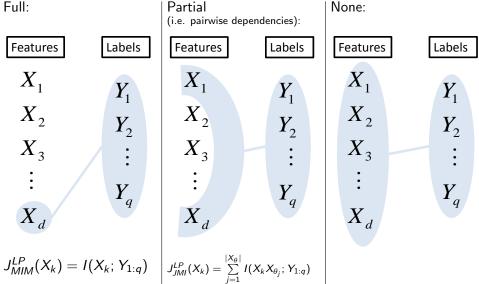


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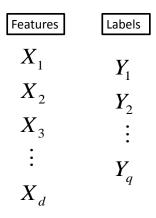
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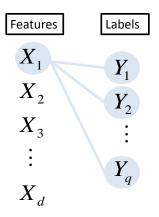
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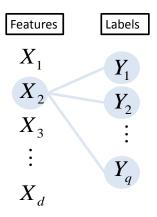
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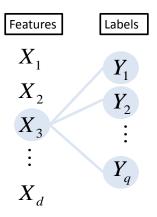
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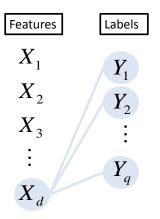
- Pros: more reliable probability estimates
- Cons: dependencies among labels are not accounted for

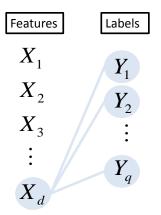






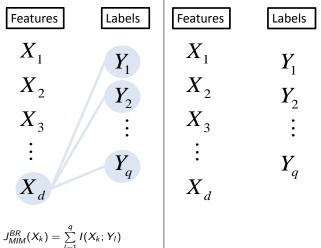




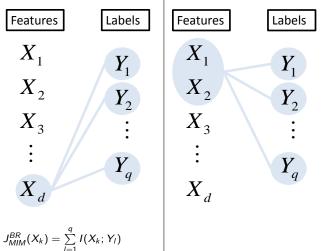


$$J_{MIM}^{BR}(X_k) = \sum_{l=1}^q I(X_k; Y_l)$$

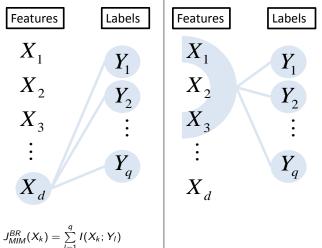
Feature space independence assumptions: Full: | Partial



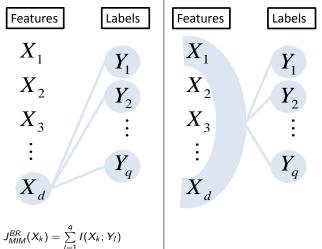
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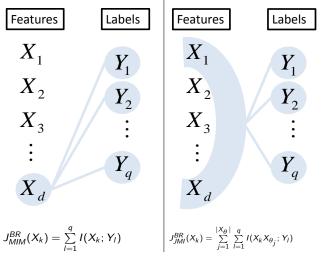
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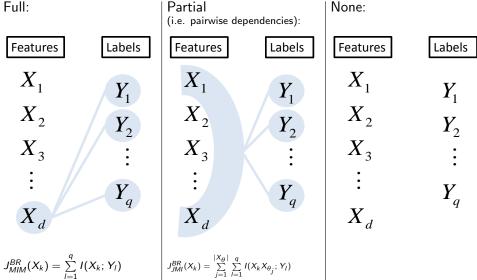


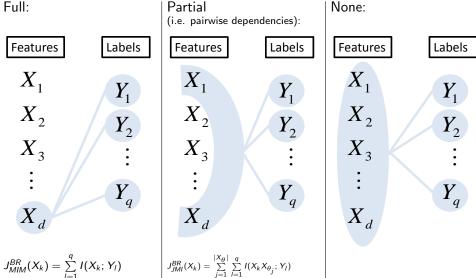
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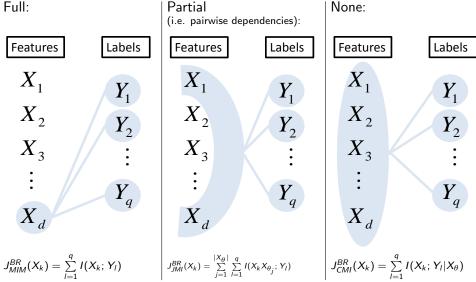
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Feature space independence assumptions:



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• Summarizing, the criteria based on feature space X and label space Y independence assumptions:

		reature space independence assumptions			
Label space		CMI (none)	JMI (partial)	MIM (full)	
independence	Label Powerset (none)	$J_{ m X:none}^{ m Y:none}$	$J_{ m X:partial}^{ m Y:none}$	$J_{ m X:full}^{ m Y:none}$	
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independence		I to the	(2013)	
				Young & Pedersen
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assumptions			-	et al. (2008),
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#### • Compare

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- effect of label space assumptions
- effect of feature space assumptions

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- our best criterion vs. state-of-the-art

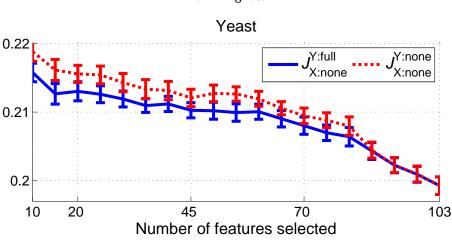
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- Evaluation: Hamming Loss (shown here) and Ranking Loss (similar)

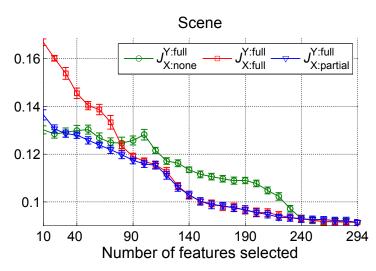
### Effect of Label Space Assumptions



Hamming loss

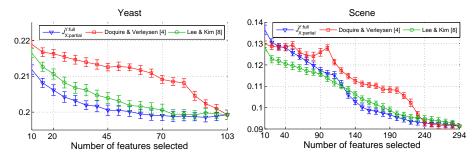
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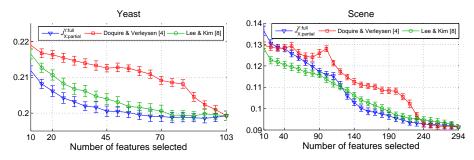


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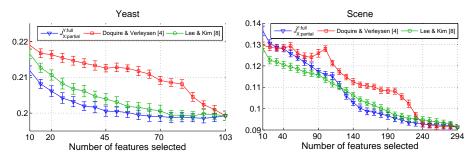






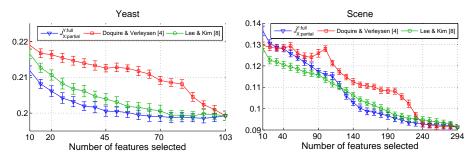
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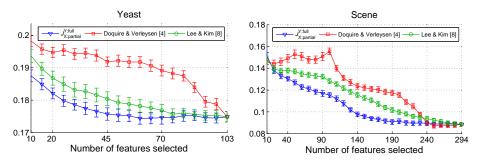
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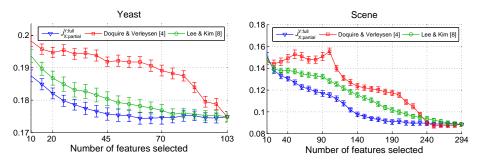
- Doquire & Verleysen (2013) :  $J_{X:none}^{Y:none}$  with pruning of rare cases
- Lee & Kim (2013) : Multivariate Mutual Information
- $J_{X:partial}^{Y:full}$  tends to outperform state-of-the-art criteria

Ranking loss



- Doquire & Verleysen (2013) :  $J_{X:none}^{Y:none}$  with pruning of rare cases
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Ranking loss



- Doquire & Verleysen (2013) :  $J_{X:none}^{Y:none}$  with pruning of rare cases
- Lee & Kim (2013) : Multivariate Mutual Information
- $J_{X:partial}^{Y:full}$  dominates state-of-the-art criteria

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  - Examining pairwise interactions seems a good compromise between capturing interdependencies vs obtaining reliable estimates...
  - ► Agrees with Brown et al. (JMLR 2012) findings in single-label filters

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- ...as informative priors P(X) or P(Y)
- ...as to how the distribution P(Y|X) is factored
- Thus constructing more problem specific filters

# Future Work: The Bigger Picture

• A typical machine learning pipeline

$$Data \xrightarrow{\mathbf{x}} \begin{array}{c} Feature \\ Selection \\ (Filter) \end{array} \xrightarrow{\mathbf{x}'} \begin{array}{c} Classification \\ (Model) \end{array} \xrightarrow{\mathbf{y}} \begin{array}{c} Evaluation \\ (Loss \ Function) \end{array}$$

# Future Work: The Bigger Picture

• A typical machine learning pipeline

$$\begin{array}{c|c} \mathsf{Feature} \\ \mathsf{Data} \xrightarrow{\mathbf{x}} & \mathsf{Feature} \\ \mathsf{Selection} \\ (Filter) & \mathsf{x'} & \mathsf{Classification} \\ (Model) & \mathsf{y} & \mathsf{Evaluation} \\ (Loss \ Function) \\ \end{array}$$

• Assumptions in every step, often conflicting...

## Future Work: The Bigger Picture

• A typical machine learning pipeline

Data 
$$\xrightarrow{\mathbf{x}}$$
 Feature  
Selection  
(Filter)  $\mathbf{x'}$  Classification  $\mathbf{y}$  Evaluation  
(Model) (Loss Function)

- Assumptions in every step, often conflicting...
- ...should investigate interplay between model, filter & loss function

# Thank you! Kiitos!