Reasoning with Expressive Description Logics: Theory and Practice

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Talk Outline

Introduction to Description Logics (DLs)

Reasoning techniques

Implementing DL systems

DL applications

- **Including demos** (time permitting)

Introduction to DLs

What are Description Logics?

- Based on **concepts** (classes) and **roles**
  - Concepts (classes) are interpreted as sets of objects
  - Roles are interpreted as binary relations on objects
- Descendants of semantic networks, frame based systems and KL-ONE
- Decidable fragments of FOL
  - Many DLs are fragments of L2, C2 or the **Guarded Fragment**
- Closely related to **propositional modal logics**
- Also known as terminological logics, concept languages, etc.
- Key features of DLs are
  - Well defined semantics (they are logics)
  - Provision of inference services

DL Applications

DLs have may applications including:

Terminological KR (including **Ontologies**)

- ☞ Medical terminology/controled vocabulary (**Galen**)
- ☞ Bio-ontologies (**Tambis**, **GO**)
- ☞ Web based ontology languages (**OIL**, **DAML+OIL**)

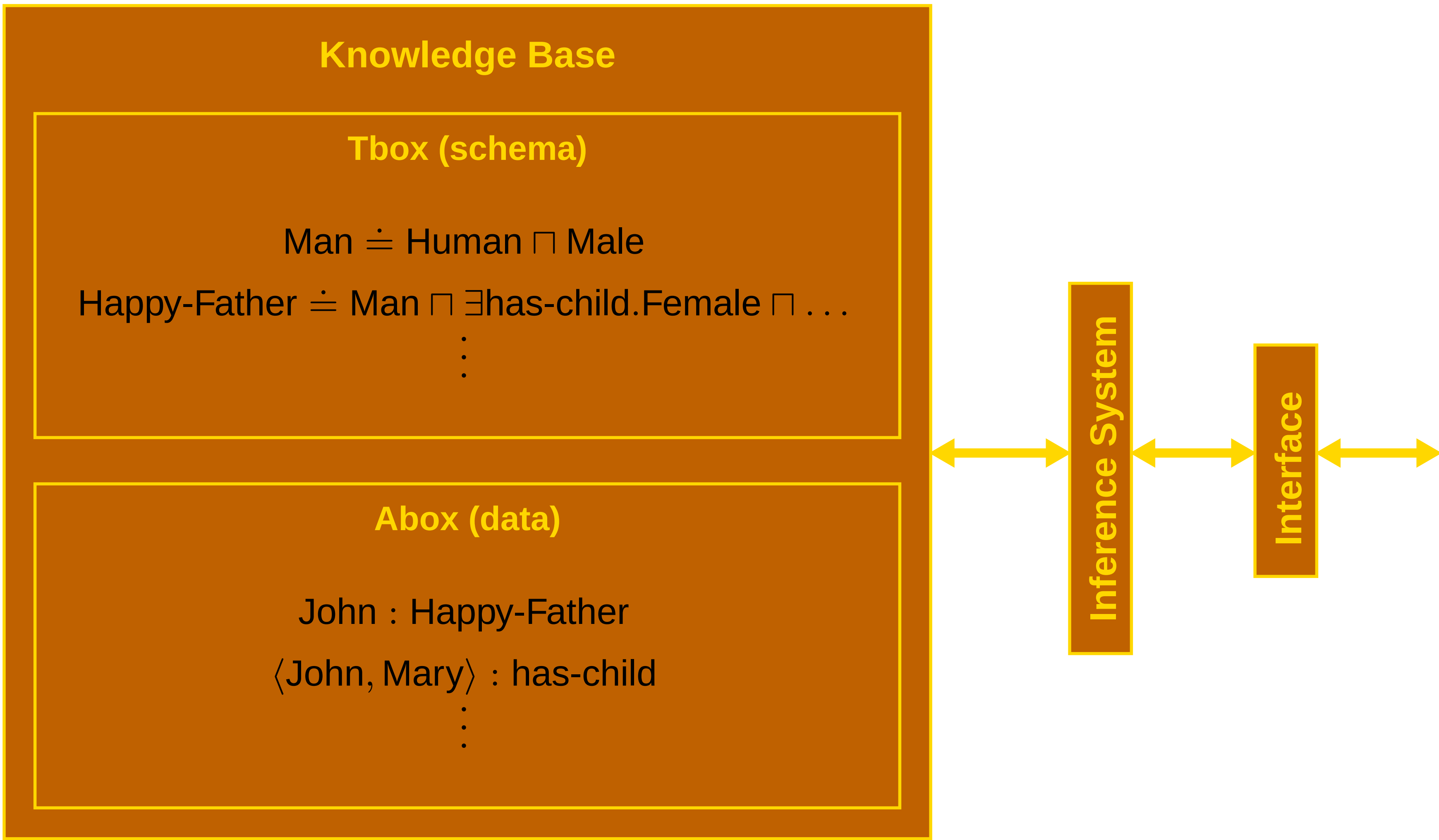
Configuration

- ☞ Classic system used to configure telecom equipment

Database schema and query reasoning

- ☞ Schema design and query optimisation
- ☞ Interoperability and federation
- ☞ Query containment (w.r.t. schema)

DL System Architecture



DL Constructors

Particular DLs characterised by **set of constructors** provided for building complex concepts and roles from simpler ones

- ☞ Usually include at least:
  - Conjunction ($\sqcap$), disjunction ($\sqcup$), negation ($\neg$)
  - Restricted (guarded) forms of quantification ($\exists$, $\forall$)
- ☞ This basic DL is known as $\mathcal{ALC}$

DL Syntax and Semantics

Semantics given by **interpretation** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

Constructor	Syntax	Example	Semantics
atomic concept	A	Human	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
atomic role	R	has-child	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

and for C, D concepts and R a role name

conjunction	$C \sqcap D$	Human $\sqcap$ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	Doctor $\sqcup$ Lawyer	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$\neg$Male	$\Delta^{\mathcal{I}} \setminus C$
exists restr.	$\exists R.C$	$\exists$has-child.Male	$\{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
value restr.	$\forall R.C$	$\forall$has-child.Doctor	$\{x \mid \forall y.\langle x, y\rangle \in R^{\mathcal{I}} \implies y \in C^{\mathcal{I}}\}$

Other DL Constructors

Many different DLs/DL constructors have been investigated, e.g.

Constructor	Syntax	Example	Semantics
number restr.	$\geqslant n R$	$\geqslant 3$ has-child	$\{x \mid \|\{y.\langle x, y\rangle \in R^{\mathcal{I}}\}\| \geqslant n\}$
	$\leqslant n R$	$\leqslant 1$ has-mother	$\{x \mid \|\{y.\langle x, y\rangle \in R^{\mathcal{I}}\}\| \leqslant n\}$
inverse role	R^{-}	has-child^{-}	$\{\langle x, y\rangle \mid \langle y, x\rangle \in R^{\mathcal{I}}\}$
trans. role	R^{*}	has-child*	$(R^{\mathcal{I}})^{*}$
concrete domain	$f_1, \ldots, f_n.P$	earns spends $<$	$\{x \mid P(f_1^{\mathcal{I}}, \ldots, f_n^{\mathcal{I}})\}$

⋮

DL Knowledge Base (Tbox)

Terminological part (**Tbox**) is set of axioms describing **structure** of domain

Definition axioms introduce macros/names for concepts

$A \doteq C$, $A \sqsubseteq C$

Father $\doteq$ Man $\sqcap$ $\exists$has-child.Human

Human $\sqsubseteq$ Animal $\sqcap$ Biped

Inclusion (GCI) axioms assert subsumption relations

$C \sqsubseteq D$ (note $C \doteq D$ equivalent to $C \doteq D$ and $D \doteq C$)

$\exists$has-degree.Masters $\sqsubseteq$ $\exists$has-degree.Bachelors

DL Knowledge Base (Abox)

Assertional part (**Abox**) is set of axioms describing **concrete situation**

Concept assertions

$a : C$

$\text{John} : \text{Man} \sqcap \exists\text{has-child}.\text{Female}$

Role assertions

$\langle a, b \rangle : R$

$\langle \text{John}, \text{Mary} \rangle : \text{has-child}$

Basic Inference Problems

Subsumption (structure knowledge, compute taxonomy)

$C \sqsubseteq D$? Is $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in all interpretations?

Subsumption w.r.t. Tbox $\mathcal{T}$

$C \sqsubseteq_{\mathcal{T}} D$? Is $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in all models of $\mathcal{T}$?

Consistency

Is C consistent w.r.t. $\mathcal{T}$? Is there a model $\mathcal{I}$ of $\mathcal{T}$ s.t. $C^{\mathcal{I}} \neq \emptyset$?

KB Consistency

Is $\langle \mathcal{T}, \mathcal{A} \rangle$ consistent? Is there a model $\mathcal{I}$ of $\langle \mathcal{T}, \mathcal{A} \rangle$?

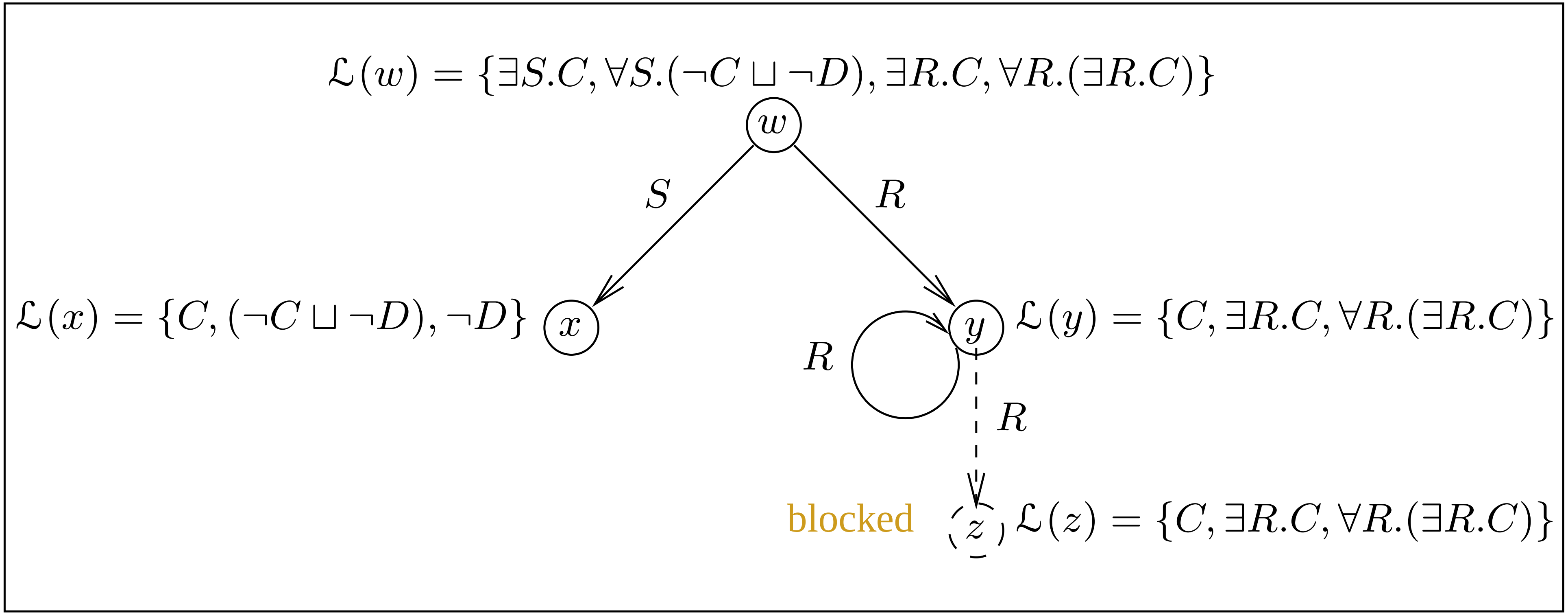
Reasoning Techniques

Subsumption and Satisfiability

- Subsumption transformed into satisfiability
- Tableaux algorithm used to test satisfiability
  - Try to build model of concept C
  - Model represented by tree $\mathbf{T}$
    - Nodes in $\mathbf{T}$ correspond to individuals in model
    - Nodes labeled with sets of subconcepts of C
    - Edges labeled with role names in C
  - Start from root node labeled $\{C\}$
  - Apply expansion rules to node labels until
    - Expansion completed (tree represents valid model)
    - Contradictions prove there is no model
  - Non-deterministic expansion $\longrightarrow$ search (e.g., $C \sqcup D$)
  - **Blocking** ensures termination (with expressive DLs)

Tableaux Expansion

Test satisfiability of $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)\}$ where R is a **transitive** role



More Advanced Techniques

Satisfiability w.r.t. a Terminology

☞ For each GCI $C \sqsubseteq D \in \mathcal{T}$, add $\neg C \sqcup D$ to **every** node label

More expressive DLs

☞ Basic technique can be extended to deal with
- Role inclusion axioms (role hierarchy)
- Number restrictions
- Inverse roles
- Concrete domains
- Aboxes

☞ Extend **expansion rules** and use more sophisticated **blocking** strategy

☞ Forest instead of Tree (for Aboxes)

Implementing DL Systems

Naive Implementations

Problems include:

- Space usage
  - Storage required for tableaux datastructures
  - Rarely a serious problem in practice
- Time usage
  - Search required due to non-deterministic expansion
  - **Serious** problem in practice
  - Mitigated by:
    - Careful **choice of algorithm**
    - Highly **optimised implementation**

Careful Choice of Algorithm

- ☞ Transitive roles instead of transitive closure
  - Deterministic expansion of $\exists R.C$, even when $R \in \mathbf{R}_+$
  - (Relatively) simple blocking conditions
  - Cycles **always** represent (part of) valid cyclical models
- ☞ Direct algorithm/implementation instead of encodings
  - GCI axioms can be used to "encode" additional operators/axioms
  - Powerful technique, particularly when used with FL closure
  - Can encode cardinality constraints, inverse roles, range/domain, . . .
    - ➜ E.g., $(\text{domain } R.C) \equiv \exists R.\top \sqsubseteq C$
  - (FL) encodings introduce (large numbers of) axioms
  - **BUT** even simple domain encoding is **disastrous** with large numbers of roles

Highly Optimised Implementation

Optimisation performed at 2 levels

- ☞ Computing **classification** (partial ordering) of concepts
  - Objective is to minimise number of subsumption tests
  - Can use standard order-theoretic techniques
    - → E.g., use **enhanced traversal** that exploits information from previous tests
  - Also use structural information from KB
    - → E.g., to select order in which to classify concepts
- ☞ Computing **subsumption** between concepts
  - Objective is to minimise cost of single subsumption tests
  - Small number of hard tests can dominate classification time
  - Recent DL research has addressed this problem (with considerable success)

Optimising Subsumption Testing

Optimisation techniques broadly fall into 2 categories

- Pre-processing optimisations
  - Aim is to **simplify KB** and facilitate subsumption testing
  - Largely algorithm independent
  - Particularly important when KB contains GCI axioms
- Algorithmic optimisations
  - Main aim is to **reduce search space** due to non-determinism
  - Integral part of implementation
  - But often generally applicable to search based algorithms

Pre-processing Optimisations

Useful techniques include

- Normalisation and simplification of concepts
  - Refinement of technique first used in $\mathcal{KRIS}$ system
  - Lexically normalise and simplify all concepts in KB
  - Combine with lazy unfolding in tableaux algorithm
  - Facilitates early detection of inconsistencies (clashes)
- Absorption (simplification) of general axioms
  - Eliminate GCIs by absorbing into "definition" axioms
  - Definition axioms efficiently dealt with by lazy expansion
- Avoidance of potentially costly reasoning whenever possible
  - Normalisation can discover "obvious" (un)satisfiability
  - Structural analysis can discover "obvious" subsumption

Normalisation and Simplification

☞ Normalise concepts to standard form, e.g.:

- $\exists R.C \longrightarrow \neg\forall R.\neg C$
- $C \sqcup D \longrightarrow \neg(\neg C \sqcap \neg D)$

☞ Simplify concepts, e.g.:

- $(D \sqcap C) \sqcap (A \sqcap D) \longrightarrow A \sqcap C \sqcap D$
- $\forall R.\top \longrightarrow \top$
- $\ldots \sqcap C \sqcap \ldots \sqcap \neg C \sqcap \ldots \longrightarrow \bot$

☞ Lazily unfold concepts in tableaux algorithm

- Use names/pointers to refer to complex concepts
- Only add structure as required by progress of algorithm
- Detect clashes between lexically equivalent concepts

Absorption I

- Reasoning w.r.t. set of GCI axioms can be very costly
  - GCI $C \sqsubseteq D$ adds $D \sqcup \neg C$ to **every** node label
  - Expansion of disjunctions leads to search
  - With 10 axioms and 10 nodes search space already 2^{100}
  - GALEN (medical terminology) KB contains **thousands** of axioms
- Reasoning w.r.t. "primitive definition" axioms is relatively efficient
  - For CN $\sqsubseteq D$, add D **only** to node labels containing CN
  - For CN $\sqsupseteq D$, add $\neg D$ **only** to node labels containing $\neg$CN
  - Can expand definitions lazily
    - Only add definitions **after** other local (propositional) expansion
    - Only add definitions one step at a time

Absorption II

- Transform GCIs into primitive definitions, e.g.
  - CN $\sqcap C \sqsubseteq D \longrightarrow$ CN $\sqsubseteq D \sqcup \neg C$
  - CN $\sqcup C \sqsupseteq D \longrightarrow$ CN $\sqsupseteq D \sqcap \neg C$
- Absorb into existing primitive definitions, e.g.
  - CN $\sqsubseteq A$, CN $\sqsubseteq D \sqcup \neg C \longrightarrow$ CN $\sqsubseteq A \sqcap (D \sqcup \neg C)$
  - CN $\sqsupseteq A$, CN $\sqsupseteq D \sqcap \neg C \longrightarrow$ CN $\sqsupseteq A \sqcup (D \sqcap \neg C)$
- Use lazy expansion technique with primitive definitions
  - Disjunctions only added to “relevant” node labels
- Performance improvements often too large to measure
  - At least **four orders of magnitude** with GALEN KB

Algorithmic Optimisations

Useful techniques include

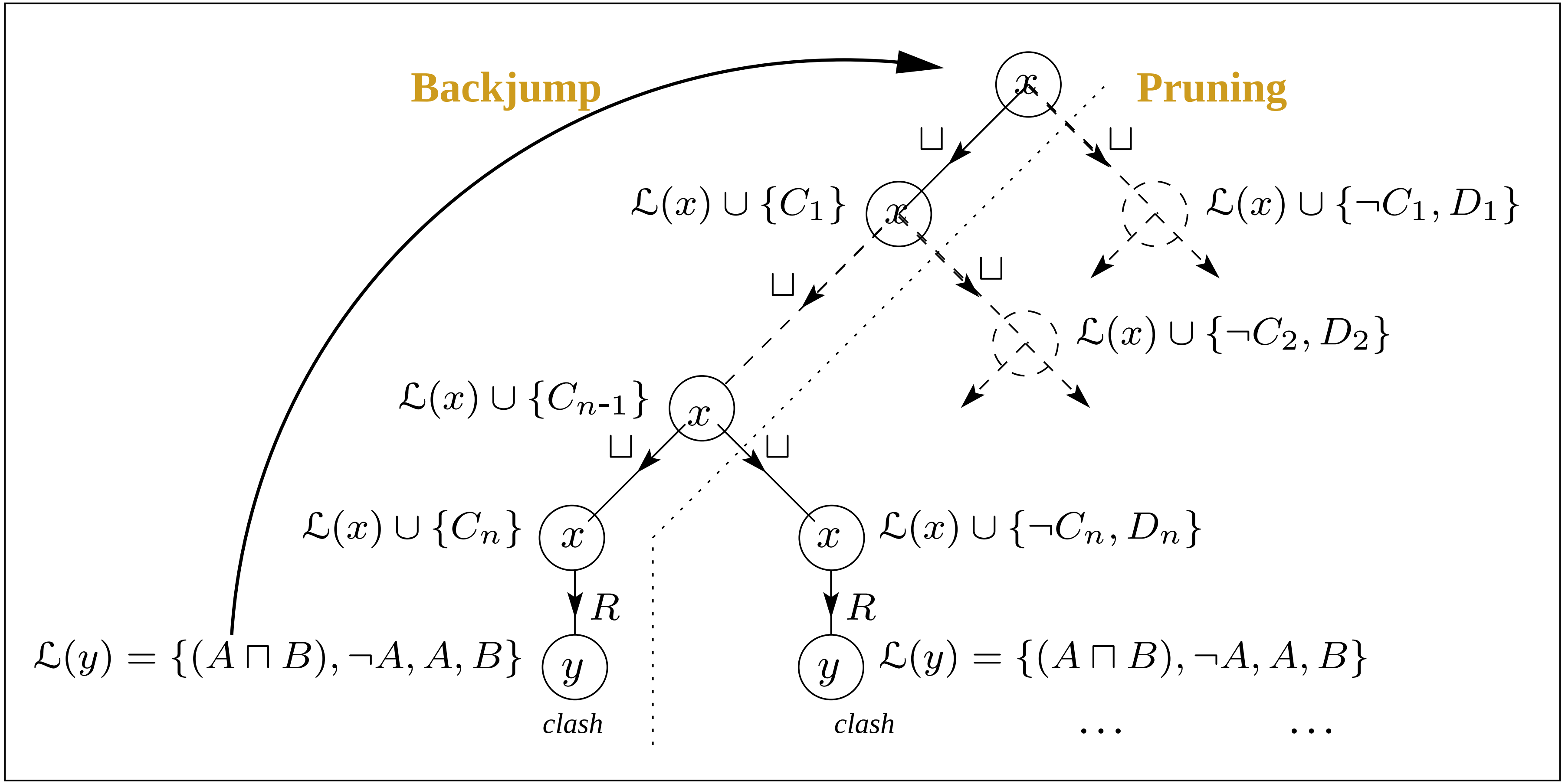
- Avoiding redundancy in search branches
  - Davis-Putnam style semantic branching search
  - Syntactic branching with no-good list
- Dependency directed backtracking
  - Backjumping
  - Dynamic backtracking
- Caching
  - Cache partial models
  - Cache satisfiability status (of labels)
- Heuristic ordering of propositional and modal expansion
  - Min/maximise constrainedness (e.g., MOMS)
  - Maximise backtracking (e.g., oldest first)

Dependency Directed Backtracking

- Allows rapid recovery from bad branching choices
- Most commonly used technique is **backjumping**
  - Tag concepts introduced at branch points (e.g., when expanding disjunctions)
  - Expansion rules combine and propagate tags
  - On discovering a clash, identify most recently introduced concepts involved
  - Jump back to relevant branch points **without exploring** alternative branches
  - Effect is to prune away part of the search space
  - Performance improvements with GALEN KB again **too large to measure**

Backjumping

E.g., if $\exists R.\neg A \sqcap \forall R.(A \sqcap B) \sqcap (C_1 \sqcup D_1) \sqcap \ldots \sqcap (C_n \sqcup D_n) \subseteq \mathfrak{L}(x)$



Caching

- Cache the satisfiability status of a node label
  - Identical node labels often recur during expansion
  - Avoid re-solving problems by caching satisfiability status
    - When $\mathcal{L}(x)$ initialised, look in cache
    - Use result, or add status once it has been computed
  - Can use sub/super set caching to deal with similar labels
  - Care required when used with blocking or inverse roles
  - Significant performance gains with some kinds of problem
- Cache (partial) models of concepts
  - Use to detect "obvious" non-subsumption
  - $C \not\sqsubseteq D$ if $C \sqcap \neg D$ is satisfiable
  - $C \sqcap \neg D$ satisfiable if models of C and $\neg D$ can be merged
  - If not, continue with standard subsumption test
  - Can use same technique in sub-problems

DL applications

Terminological KR and Ontologies

- General requirement for medical terminologies
- Static lists/taxonomies difficult to build and maintain
  - Need to be very large and highly interconnected
  - Inevitably contain many errors and omissions
- Galen project replaced static hierarchy with DL
  - Describe concepts (e.g., spiral fracture of left femur)
  - Use DL classifier to build taxonomy
- Needed expressive DL **and** efficient reasoning
  - Descriptions used transitive roles, inverses, GCIs etc.
  - Even prototype KB was very large ($\approx$3,000 concepts)
  - Existing (incomplete) classifier took $\approx$24 hours to classify KB
  - FaCT system (sound and complete) takes $\approx$60s

The Semantic Web

- Most existing Web resources only human understandable
  - Markup (HTML) provides rendering information
  - Textual/graphical information for human consumption
  - Semantic Web aims at machine understandability
    - **Semantic** markup will be added to web resources
    - Markup will use **Ontologies** for shared understanding
    - Requirement for DAML ontology language
    - Should extend existing Web standards (XML, RDF, RDFS)

OIL and DAML+OIL

☞ Intuitive (frame) syntax plus high expressive power

☞ Well defined semantics via mapping to $\mathcal{SHIQ}$ DL

☞ Can use FaCT system to reason with OIL ontologies

☞ Extends existing Web standards (XML, RDF, RDFS)

☞ Effectively a DL with RDFS based syntax

☞ Can use DL reasoning with DAML+OIL

☞ E.g., **OilEd** ontology editor
  - Frame based interface (e.g., Protegé, OntoEdit)
  - Extended to capture whole of OIL/DAML+OIL languages
  - Reasoning support from FaCT (via CORBA interface)

OilEd

E.g., DAML+OIL medical terminology ontology

- ☞ Transitive roles capture transitive partonomy, causality, etc.

  $\text{Smoking} \sqsubseteq \exists \text{causes.Cancer}$ plus $\text{Cancer} \sqsubseteq \exists \text{causes.Death}$

  $\Rightarrow \text{Cancer} \sqsubseteq \text{FatalThing}$

- ☞ GCIs represent additional non-definitional knowledge

  $\text{Stomach-Ulcer} \doteq \text{Ulcer} \sqcap \exists \text{hasLocation.Stomach}$ plus

  $\text{Stomach-Ulcer} \sqsubseteq \exists \text{hasLocation.Lining-Of-Stomach}$

  $\Rightarrow \text{Ulcer} \sqcap \exists \text{hasLocation.Stomach} \sqsubseteq \text{OrganLiningLesion}$

- ☞ Inverse roles capture e.g. cases/causedBy relationship

  $\text{Death} \sqcap \exists \text{causedBy.Smoking} \sqsubseteq \text{PrematureDeath}$

  $\Rightarrow \text{Smoking} \sqsubseteq \text{CauseOfPrematureDeath}$

- ☞ Cardinality restrictions add consistency constraints

  $\text{BloodPressure} \sqsubseteq \exists \text{hasValue.}(\text{High} \sqcup \text{Low}) \sqcap \leqslant 1 \text{hasValue}$ plus

  $\text{High} \sqsubseteq \neg \text{Low} \Rightarrow \text{HighLowBloodPressure} \sqsubseteq \bot$

Database Schema and Query Reasoning

- $\mathcal{DLR}$ (n-ary DL) can capture semantics of many datamodelling methodologies (e.g., EER)
- Satisfiability preserving mapping to $\mathcal{SHIQ}$ allows use of DL reasoners (e.g., FaCT, RACER)
- DL Abox can also capture semantics of conjunctive queries
  - Can reason about query containment w.r.t. schema
- DL reasoning can be used to support, e.g.
  - Schema design and integration
  - Query optimisation
  - Interoperability and federation
- E.g., **I.COM** Intelligent Conceptual Modelling tool (Enrico Franconi)
  - Uses FaCT system to provide reasoning support for EER

Summary

- ☞ DLs are **logic based** KR formalisms
- ☞ DL systems provide **efficient inference services**
  - Careful choice of logic/algorithm
  - Highly optimised implementation
- ☞ Have proved effective in a range of applications
  - Terminologies/Ontologies
  - Databases
- ☞ Have been influential in development of Semantic Web
  - Web standard ontology language will be DL based

Resources

Slides from this talk

`www.cs.man.ac.uk/~horrocks/Slides/leipzig-jun-01.pdf`

FaCT system

`www.cs.man.ac.uk/fact`

OIL

`www.ontoknowledge.org/oil/`

DAML+OIL

`www.daml.org/language/`

OilEd

`img.cs.man.ac.uk/oil`

I.COM

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