Tableau rules for $\mathcal{ALC}$

$\rightarrow_{\sqcap}$: **If** $C \sqcap D \in \mathcal{L}(x)$ **but** $\{C, D\} \cap \mathcal{L}(x) = \emptyset$
Then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C, D\}$

$\rightarrow_{\sqcup}$: **If** $C \sqcup D \in \mathcal{L}(x)$ **but** $\{C, D\} \not\subseteq \mathcal{L}(x)$
Then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{E\}$ **for** $E \in \{C, D\}$

$\rightarrow_{\exists}$: **If** $\exists R.C \in \mathcal{L}(x)$ **but** x **has no** R**-successor** y **with** $C \in \mathcal{L}(y)$
Then **create new** R**-successor** y **of** x **with** $\mathcal{L}(y) = \{C\}$

$\rightarrow_{\forall}$: **If** $\forall R.C \in \mathcal{L}(x)$ **and** x **has an** R**-successor** y **with** $C \notin \mathcal{L}(y)$
Then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{C\}$

Tableau rules for $\mathcal{ALC}$ with GCIs

$$\{C_i \dot{\sqsubseteq} D_i \mid 1 \leq i \leq n\}$$

applicable only to nodes x that are not blocked:

y **is blocked by an ancestor** x **if** $\mathfrak{L}(y) \subseteq \mathfrak{L}(x)$

$\rightarrow_{\sqcap}$: **If** $C \sqcap D \in \mathfrak{L}(x)$ **but** $\{C, D\} \cap \mathfrak{L}(x) = \emptyset$
Then $\mathfrak{L}(x) \rightarrow \mathfrak{L}(x) \cup \{C, D\}$

$\rightarrow_{\sqcup}$: **If** $C \sqcup D \in \mathfrak{L}(x)$ **but** $\{C, D\} \not\subseteq \mathfrak{L}(x)$
Then $\mathfrak{L}(x) \rightarrow \mathfrak{L}(x) \cup \{E\}$ **for** $E \in \{C, D\}$

$\rightarrow_{\exists}$: **If** $\exists R.C \in \mathfrak{L}(x)$ **but** x **has no** R**-successor** y **with** $C \in \mathfrak{L}(y)$
Then **create new** R**-successor** y **of** x **with** $\mathfrak{L}(y) = \{C\}$

$\rightarrow_{\forall}$: **If** $\forall R.C \in \mathfrak{L}(x)$ **and** x **has an** R**-successor** y **with** $C \notin \mathfrak{L}(y)$
Then $\mathfrak{L}(y) \rightarrow \mathfrak{L}(y) \cup \{C\}$

$\rightarrow_{\mathrm{GCI}}$: **If** $(\neg C_i \sqcup D_i) \notin \mathfrak{L}(x)$ **for some** $1 \leq i \leq n$
Then $\mathfrak{L}(x) \rightarrow \mathfrak{L}(x) \cup \{\neg C_i \sqcup D_i\}$

Tableau rules for $\mathcal{ALCI}$ with GCIs

$$\{C_i \dot{\sqsubseteq} D_i \mid 1 \leq i \leq n\}$$

applicable only to nodes x that are not blocked:

y **is blocked by an ancestor** x **if** $\mathcal{L}(x) = \mathcal{L}(y)$

$\rightarrow_{\sqcap}$: **If** $C \sqcap D \in \mathcal{L}(x)$ **but** $\{C, D\} \cap \mathcal{L}(x) = \emptyset$
Then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C, D\}$

$\rightarrow_{\sqcup}$: **If** $C \sqcup D \in \mathcal{L}(x)$ **but** $\{C, D\} \not\subseteq \mathcal{L}(x)$
Then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{E\}$ **for** $E \in \{C, D\}$

$\rightarrow_{\exists}$: **If** $\exists R.C \in \mathcal{L}(x)$ **but** x **has no** R**-neighbour** y **with** $C \in \mathcal{L}(y)$
Then **create new** R**-successor** y **of** x **with** $\mathcal{L}(y) = \{C\}$

$\rightarrow_{\forall}$: **If** $\forall R.C \in \mathcal{L}(x)$ **and** x **has an** R**-neighbour** y **with** $C \notin \mathcal{L}(y)$
Then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{C\}$

$\rightarrow_{\text{GCI}}$: **If** $(\neg C_i \sqcup D_i) \notin \mathcal{L}(x)$ **for some** $1 \leq i \leq n$
Then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C_{\mathcal{T}}\}$

Additional tableau rules for $\mathcal{ALCQI}$ with GCIs applicable only to nodes x that are not blocked:

y is blocked by an ancestor y' if there are x, x' with

- **y is succ. of x, y' is succ. of x',**
- **$\mathfrak{L}(x) = \mathfrak{L}(y)$, $\mathfrak{L}(x') = \mathfrak{L}(y')$, and**
- **$\mathfrak{L}(\langle x, y\rangle) = \mathfrak{L}(\langle x', y'\rangle)$.**

$\rightarrow_{\geq}$: **If** $(\geqslant nR.C) \in \mathfrak{L}(x)$, **$x$ is not blocked, and x has less than n R-neighbours y_i with $C \in \mathfrak{L}(y_i)$**
Then **create n new R-successor $y_1, \ldots, y_n$ of x with $\mathfrak{L}(y_i) := \{C\}$ and $y_i \dot{\neq} y_j$ for all $i \neq j$**

$\rightarrow_{\leq}$: **If** $(\leqslant nR.C) \in \mathfrak{L}(x)$, **$x$ is not indirectly blocked, x has $n+1$ R-neighbours $y_0, \ldots, y_n$ with $C \in \mathfrak{L}(y_i)$, and there are i, j with** *not* **$y_i \dot{\neq} y_j$ and y_j is** *not* **an ancestor of y_i**
Then $\mathfrak{L}(y_i) \rightarrow \mathfrak{L}(y_i) \cup \mathfrak{L}(y_j)$,
make y_j's successors to successors of y_i,
add $y_i \dot{\neq} z$ for each z with $y_j \dot{\neq} z$,
remove y_j from the tree

$\rightarrow_{choice}$: **If** $(\leqslant nR.C) \in \mathfrak{L}(x)$, **$x$ is not indirectly blocked, x has an R-neighbour y with** $\{C, \dot{\neg} C\} \cap \mathfrak{L}(y) = \emptyset$
Then $\mathfrak{L}(y) \rightarrow \mathfrak{L}(y) \cup \{D\}$ **for some** $D \in \{C, \dot{\neg} C\}$

Translation of $\mathcal{ALCQIO}$-concepts into C2

(The mapping t_y is obtained by switching the roles of x and y in t_x)

$$
\begin{aligned}
t_x(A) &= \mathbf{A}(x),\\
t_x(\neg C) &= \neg t_x(C)\\
t_x(C \sqcap D) &= t_x(C) \wedge t_x(D),\\
t_x(C \sqcup D) &= t_x(C) \vee t_x(D),\\
t_x(\exists R.C) &= \exists y.\mathbf{R}(x,y) \wedge t_y(C)\\
t_x(\forall R.C) &= \forall y.\neg\mathbf{R}(x,y) \vee t_y(C)\\
t_x({\geqslant} nR.C) &= \exists^{\geq n} y.\mathbf{R}(x,y) \wedge t_y(C),\\
t_x({\geqslant} nR^-.C) &= \exists^{\geq n} y.\mathbf{R}(y,x) \wedge t_y(C),\\
t_x({\leqslant} nR.C) &= \exists^{\leq n} y.\mathbf{R}(x,y) \wedge t_y(C),\\
t_x({\leqslant} nR^-.C) &= \exists^{\leq n} y.\mathbf{R}(y,x) \wedge t_y(C)
\end{aligned}
$$

$$
t(\mathcal{T}) = \bigwedge_{C \dot{\sqsubseteq} D \in \mathcal{T}} \forall x.t_x(C) \Rightarrow t_x(D)
$$

$$
\begin{aligned}
t(R \dot{\sqsubseteq} S) &= \forall x,y.\mathbf{R}(x,y) \Rightarrow \mathbf{S}(x,y)\\
t(\mathsf{trans}(R)) &= \forall x,y,z.(\mathbf{R}(x,y) \wedge \mathbf{R}(y,z)) \Rightarrow \mathbf{R}(x,z)\\
t_x(o) &= (x = a_o),\ \text{for nominal } o \text{ and constant } a_o
\end{aligned}
$$

Lemma:

1. $\mathsf{sat}(C,\mathcal{T})$ **iff** $t_x(C) \wedge t(\mathcal{T})$ **is satisfiable**

2. $\mathsf{sat}(C,D,\mathcal{T})$ **iff** $t(\mathcal{T}) \Rightarrow (\forall x.t_x(C) \Rightarrow t_x(D)$