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Sequential reflexive logics with a noncontingency operator. (Russian. Russian summary)
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Propositional logics with modal operator $\rhd$ (noncontingency), understood as $\rhd A \leftrightharpoons \Box A \vee \Box\neg A$, are in focus. This definition gives a translation $\rhd\colon A \mapsto A\rhd$ from the $\rhd$-language into the $\Box$-language. Now, if L is a modal logic in the $\Box$-language, the logic of noncontingency over L, $L^{\rhd}$, is all the $\rhd$-formulas, $\rhd$-images of which are valid in L. It is easily seen that, if L contains $\Box A \to A$, then $\Box A$ is equivalent in L to $A \& \rhd A$. It suggests the following axiomatization of $L^{\rhd}$ for an extension $L = \mathrm{T} + \Gamma$ of the modal logic T: $L^{\rhd} = \mathrm{T}^{\rhd} + \mathrm{Tr}(\Gamma)$, where Tr is the translation from $\Box$-language into $\rhd$-language that skips variables, commutes with Boolean connectives, and with: $\mathrm{Tr}(\Box A) = \mathrm{Tr}(A) \& \rhd \mathrm{Tr}(A)$ (Lemma 4.5). The author has considered axiomatizations for $\mathrm{K}^{\rhd}$, $\mathrm{K4}^{\rhd}$ and $\mathrm{GL}^{\rhd}$ elsewhere [Vestnik Moskov. Univ. Ser. I Mat. Mekh. **2001**, no. 6, 43–48, 65; MR1890172 (2002k:03031)].

In the present paper, the author's attention is devoted to sequential calculi for $L^{\rhd}$, where L is one of T, $\mathrm{S4}$, B, $\mathrm{S5}$ or Grz. Through Kripke-style semantics, it is proved that the proposed sequential calculi are complete with respect to their Hilbert-style counterparts (Theorem 4.1). Although cut cannot be eliminated in any of the proposed sequential calculi, all of them enjoy the Craig interpolation property. This is a consequence of the more general statement: $L^{\rhd}$, where L is an extension of T, has the interpolation property if and only if L has it (Lemma 5.3).

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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