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Zolin, E. E.
Sequential logic of arithmetic decidability. (Russian. Russian summary)
Vestnik Moskov. Univ. Ser. I Mat. Mekh. **2001,** *no. 6,* 43–48, 65; *translation in* *Moscow Univ. Math. Bull.* **56** (2001), *no. 6,* 22–27 (2002).
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References: 0	**Reference Citations: 1**	**Review Citations: 1**

If $\Box A$ means "A is derivable in arithmetic", then $\triangleright A = \Box A \vee \Box\neg A$ means "A is formally decidable". Based on previous axiomatizations of the non-contingency connective in the modal logics K, K4, S4 and the axiomatization of provability logic by Solovay, the author axiomatizes the provability logic of $\triangleright$. It has the following axioms:

Tautologies; $\triangleright p \to \triangleright\neg p$; $\triangleright(p \leftrightarrow q) \to (\triangleright p \leftrightarrow \triangleright q)$; $\triangleright p \to [\triangleright(q \to p) \vee \triangleright(p \to r)]$; $\triangleright p \to \triangleright(q \to \triangleright p)$; $\triangleright(\triangleright p \to p) \to \triangleright p$.

The rules are modus ponens and $A/\triangleright A$. Completeness with respect to finite irreflexive transitive frames is proved by constructing a canonical model of complete sequents consisting of subformulas of a given formula. $\Box A$ is approximated by a set $\{\triangleright(B \vee A)\colon B$ belongs to a suitable set$\}$. The accessibility relation $x \downarrow w$ between the worlds $x = (\Gamma \to \Delta)$ and $w = (\Gamma' \to \Delta')$ is defined in such a way that $\triangleright A \in \Gamma'$ iff $(\forall x \downarrow w\ A \in \Gamma)$ or $(\forall x \downarrow w\ A \in \Delta)$.

Reviewed by *G. E. Mints*