Firstly, I have noticed that, in the paper, I started the proof of Theorem 5.8 by saying:
We prove a bit more: if $L \subseteq M(\nabla)$ for some modality ∇ then $\mathbf{\Lambda}_{\bigcirc\top} \subseteq M(\nabla)$.
But, in fact, I have proved exactly what is claimed in Theorem 5.8. Because, in step (g), I use the fact that $M(\nabla)$ is closed under the rule (Nec), which follows from the equality $L = M(\nabla)$, but not from the inclusion $L \subseteq M(\nabla)$ only.

In the proof of Theorem 5.8 (and already implicitly in Definition 5.4 of a modalised logic), I use the following fact, widely used in Discrete mathematics (when they build circuits out of elements computing $\wedge, \vee, \neg$ to compute boolean functions).

Fact. Suppose that we have a boolean function $f(\vec{x}, \vec{y})\colon \mathbf{2}^{m+n} \to \mathbf{2}$, where $\mathbf{2} = \{\bot, \top\}$, $\vec{x} = (x_1, \ldots, x_m)$, $\vec{y} = (y_1, \ldots, y_n)$. Then we have the *decomposition of the function* $f(\vec{x}, \vec{y})$ *w.r.t. the variables* $\vec{x}$:

$$f(\vec{x}, \vec{y}) \longleftrightarrow \bigvee_{\vec{\sigma} \in \mathbf{2}^m} \left(\vec{x}^{\,\vec{\sigma}} \wedge f(\vec{\sigma}, \vec{y}) \right)$$

where $\vec{\sigma} = (\sigma_1, \ldots, \sigma_m)$, $\vec{x}^{\,\vec{\sigma}} = x_1^{\sigma_1} \wedge \ldots \wedge x_m^{\sigma_m}$, $x^\top = x$, $x^\bot = \neg x$. In particular, if $\vec{y}$ is an empty tuple, we have a Full DNF for $f(\vec{x})$.
Indeed, if we compute the value of the left and right hand sides on any tuples $\vec{x} = \vec{\sigma}$, $\vec{y} = \vec{\tau}$, then on the L.H.S. we have $f(\vec{\sigma}, \vec{\tau})$, where as on the R.H.S. all the disjuncts are false except for the only one: $\vec{\sigma}^{\,\vec{\sigma}} \wedge f(\vec{\sigma}, \vec{\tau})$, which is equivalent to $f(\vec{\sigma}, \vec{\tau})$.
Moreover, we have the following **rule**: in order to compute the formula that stands in conjunction with $\vec{x}^{\vec{\sigma}}$, we simply substitute $\vec{\sigma}$ for $\vec{x}$ in f, thus obtaining $f(\vec{\sigma}, \vec{y})$.

For example, in Definition 5.4, we take any modal formula A, consider it as a boolean function of its variables $\vec{p}$ and boxed formulas:

$$A \;\leftrightarrow\; f(\vec{p}, \Box F_1, \ldots, \Box F_n),$$

and then apply the above decomposition w.r.t. the variables $\vec{p}$ (neglecting the fact that the variables $\vec{p}$ may also occur in the formulas F_i):

$$A \longleftrightarrow \bigvee_{\vec{\sigma} \in \mathbf{2}^m} \left(\vec{p}^{\,\vec{\sigma}} \wedge B_{\vec{\sigma}} \right). \qquad (\star)$$

In order to compute $B_{\vec{\sigma}}$, we can apply the above **rule** and substitute $\vec{\sigma}$ for non-modalised occurrences of the variables $\vec{p}$ in A. This will be used in the proof below.

To keep the whole thing on one page, I continue overleaf.

Theorem 5.8. *Suppose a logic $L \supseteq$ **KB** is normal, a logic $M \supseteq$ **E** is modalised, and $L \hookrightarrow M$. Then $L \supseteq$ (**Triv** $\cap$ **Ver**).*

Proof (Extended). Firstly, we decompose the formula ∇p w.r.t. the variable p:

$$\nabla p \longleftrightarrow [(p \wedge \Delta p) \vee (\neg p \wedge \Delta' p)],$$

where the formulas Δp and $\Delta' p$ are modalised.

Next, we recall that M proves the distributivity principle for ∇, since $L \hookrightarrow M$:

$$M \vdash \nabla(p \to q) \to (\nabla p \to \nabla q)$$

Denote this formula by D, then substitute the decompositions of ∇p, ∇q, and $\nabla(p \to q)$ into D, thus obtaining an equivalent formula. It is a boolean combination of the variables p, q and some boxed formulas. Now take a decomposition of D w.r.t. the variables p, q, which looks like:

$$\begin{array}{rcl} D & \longleftrightarrow & (\neg p \wedge \neg q \wedge \mathsf{(a)}) \vee \\ & & (\neg p \wedge \quad q \wedge \mathsf{(b)}) \vee \\ & & (p \wedge \neg q \wedge \mathsf{(c)}) \vee \\ & & (p \wedge \quad q \wedge \mathsf{(d)}) \end{array}$$

To compute the formulas (a)–(d), simply apply the **rule** from the above mentioned **Fact**. For example, to compute the formula (b), which stands in conjunction with $\neg p \wedge q$, we substitute $p := \bot$ and $q := \top$ into D (of course, we substitute only for non-modalised occurrences of p and q, i.e., that are not in scope of $\Box$'s). Now we recall that the formula ∇r turns into Δr, if we substitute $r := \top$ for non-modalised occurrences of r, and it turns into $\Delta' r$ if we substitute $r := \bot$.

Therefore, under the substitution considered, ∇p will turn into $\Delta' p$, ∇q into Δq, and $\nabla(p \to q)$ into $\Delta(p \to q)$, just because the implication $(p \to q)$ is true for these values of p and q. Thus, under this substitution, the formula D turns into the following formula (b):

$$\mathsf{(b)} \quad \Delta(p \to q) \to (\Delta' p \to \Delta q).$$

By the way, that is why in the four formulas that we obtain in this process:

$$\begin{array}{ll} \mathsf{(a)} & \Delta(p \to q) \to (\Delta' p \to \Delta' q) \\ \mathsf{(b)} & \Delta(p \to q) \to (\Delta' p \to \Delta q) \\ \mathsf{(c)} & \Delta'(p \to q) \to (\Delta p \to \Delta' q) \\ \mathsf{(d)} & \Delta(p \to q) \to (\Delta p \to \Delta q) \end{array}$$

the column of Delta's marked in blue resembles the truth table for implication, and the remaining two columns of Delta's look like the truth tables for p and q, resp.

Now it remains to note that since M is a modalised logic and $M \vdash D$, it proves all the four formulas (a)–(d) (they play the role of $B_{\vec{\sigma}}$ from Definition 5.4 of a modalised logic).

Next we deal with the symmetry $p \to \Box\Diamond p$ ... Again, let's go overleaf.

Next we deal with the symmetry $p \to \Box\Diamond p$ in a similar way: since $L \hookrightarrow M$, we have

$$M \vdash p \to \nabla\neg\nabla\neg p.$$

Let us first decompose the outermost Nabla, i.e., the formula ∇F, where $F := \neg\nabla\neg p$. We have:

$$\nabla F \;\longleftrightarrow\; [(F \wedge \Delta F) \vee (\neg F \wedge \Delta' F)].$$

So, we substitute this equivalence to the formula $p \to \nabla\neg\nabla\neg p$ and obtain:

$$p \to [(\neg\nabla\neg p \wedge \Delta\neg\nabla\neg p) \vee (\nabla\neg p \wedge \Delta'\neg\nabla\neg p)]. \tag{\#}$$

Note that two new outermost Nablas appeared now, so we have to decompose them too (but see the **Remark** below). That is, we replace both outermost formulas $\nabla\neg p$ (marked in blue) with longer ones, using the equivalence:

$$\nabla\neg p \;\longleftrightarrow\; \{(\neg p \wedge \Delta\neg p) \vee (p \wedge \Delta'\neg p)\}.$$

This results in the following formula (way too long to fit one line):

$$\begin{aligned} p \to \big[&(\neg\{(\neg p \wedge \Delta\neg p) \vee (p \wedge \Delta'\neg p)\} \wedge \Delta\neg\nabla\neg p) \vee \\ &(\{(\neg p \wedge \Delta\neg p) \vee (p \wedge \Delta'\neg p)\} \wedge \Delta'\neg\nabla\neg p)\big]. \end{aligned} \tag{$\flat$}$$

Now denote this formula by S and apply decomposition: $S \leftrightarrow ((p \wedge S_1) \vee (\neg p \vee S_2))$. Recall our **rule**: in order to compute S_1 (which stands in conjunction with p), we substitute $p := \top$ for all *non-modalized* occurrences of p in S, thus obtaining:

$$(\neg\Delta'\neg p \wedge \Delta\neg\nabla\neg p) \;\vee\; (\Delta'\neg p \wedge \Delta'\neg\nabla\neg p). \tag{S_1}$$

Next, in order to compute S_2, we substitute $p := \bot$ for all *non-modalized* occurrences of p in S, thus obtaining a tautology: $S_2 = \top$, since p stands as the antecedent (so, this fact is not an obstacle, but, on the contrary, it makes our job easier).

Since $M \vdash S$ and M is a modalised logic, we infer $M \vdash S_1$ (and $M \vdash S_2$, but this is trivial). Observe that S_1 has the form of disjunction; and logicians usually do not like theorems in the form of disjunction (do they?). Therefore, we go a step further: notice that S_1 has the form $(\neg A \wedge B) \vee (A \wedge C)$. It is truth-functionally equivalent to the following *conjunction*: $(A \to C) \wedge (A \vee B)$ (just build two truth tables and compare them). Hence, M proves the conjunction:

$$M \vdash \;(\Delta'\neg p \to \Delta'\neg\nabla\neg p) \;\wedge\; (\Delta'\neg p \vee \Delta\neg\nabla\neg p).$$

Hence M proves both conjuncts. It remains to simplify them by replacing $\neg p$ with p to obtain:

(e) $\Delta' p \to \Delta'\neg\nabla p$;
(f) $\Delta' p \vee \Delta\neg\nabla p$.

That's it!

Remark. In fact, the proof I carried out in my drafts is much shorter (but I decided to present the long one, since it exactly corresponds to the steps on p. 880 of the paper). Namely, before transforming (#) into a long-long formula ($\flat$), recall that later we will need to compute S_1 and S_2 anyway; and already at the stage (#) we can observe that S_2 will be a tautology. So it remains to compute S_1.

To this end, we subsitute $p := \top$ for all non-modalized occurrences of p already in (#). The antecedent p disappears, whereas the two blue subformulas $\nabla\neg p$ are equivalent (under this substitution $p := \top$) to $\Delta'\neg p$. Thus, we avoid ($\flat$) and directly come to the formula marked as (S_1).

As always, I would be glad to provide any further details.